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ABSTRACT

Prior research on problem solving is discussed, followed by a review of research on mental processes, intellectual development, information processing, mental images, and conceptual imagery. Then four intact fourth grade classes of 23 pupils each were assigned to a 2 x 2 factorial design. One factor was the type of instruction (Method A and Method B); the other factor consisted of two teachers. Method A classes received instruction in the pupil generation of diagrams to represent the inherent structure of mathematics word problems. Method B pupils were taught an eclectic approach. After eight hours of differential instruction an analysis of covariance, using five pretreatment measures as covariates, showed the Method A pupils to be superior in word problem skills ($p < .001$) and attitudes toward mathematics ($p < .02$). They demonstrated the same superiority six weeks after instruction. The Method A pupils generating the greater number of correct diagrams on the posttest and retention test also performed significantly better on those tests. The efficacy of this new teaching strategy was demonstrated and its potential for diagnosing pupil mental processes was indicated. (Author/MNS)

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PUPIL GENERATED DIAGRAMS AS A STRATEGY FOR SOLVING WORD PROBLEMS IN ELEMENTARY MATHEMATICS

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By

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B.S., Elizabethtown College, 1969
M.Ed., The Pennsylvania State University, 1972

A Thesis
Submitted to the Faculty of the
Graduate School of the University of Louisville
in Partial Fulfillment of the Requirements
for the Degree of

Specialist in Education

Department of Early and Middle Childhood Education
University of Louisville
Louisville, Kentucky

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Four intact fourth grade classes of 23 pupils each were assigned to a 2 x 2 factorial design. One factor was the type of instruction (Method A and Method B); the other factor consisted of two teachers. Method A classes received instruction in the pupil generation of diagrams to represent the inherent structure of mathematics word problems. Method B pupils were taught an eclectic approach. After eight hours of differential instruction an analysis of covariance, using five pretreatment measures as covariates, showed the Method A pupils to be superior in word problem skills ($p < .001$) and attitudes towards mathematics ($p < .02$). They demonstrated the same superiority six weeks after instruction. The Method A pupils generating the greater number of correct diagrams on the posttest and retention test also performed significantly better on those tests. The efficacy of this new teaching strategy was demonstrated and its potential for diagnosing pupil mental processes was indicated.

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CHAPTER I

LITERATURE REVIEW

Dr. Tommie A. West (1977) has stated--in the words of elementary teachers--why there has been so much research in the area of verbal problem solving:

> Why can't children solve verbal problems? "They can't read," respond some teachers. "They can't think," reply others. "They are careless," say some. "They can't decide what to do," answer still others. Although these reasons may be true, they do not adequately describe why so many children fail to solve verbal problems, and they do not give much direction for improving problem solving ability [p. 57].

Verbal problem solving is here defined as the independent translation of a word problem into appropriate number sentences and accurately performing the associated algebra and/or arithmetic involved. All researchers agree that problem solving is the most difficult area of teaching/learning in elementary mathematics (Suydam & Weaver, 1977). An article in TIME magazine (1979) reported that a national survey had confirmed this fact. This article summarized the results of the 1978 National Institute of Education's publication, National Assessment of Educational Progress (1979). Word problems were clearly the most difficult for the 71,000 students tested. A similar report of this study (Louisville Times, 1979) also noted that the 1980 Yearbook of the National Council of Teachers of Mathematics Problem Solving in School Mathematics, focuses primarily upon this area of mathematics instruction.

The Louisville Times article (1979) also noted that, unfortunately, the popular "back to basics" movement will likely decrease rather than increase teacher emphasis upon this most difficult area of mathematics

instruction. This will be largely due to the "back to basics" emphasis upon the rote memory of drill and practice rather than the understanding necessary for solving problems. This is especially ironic in view of the fact that the "back to basics" advocates charge that contemporary students can not deal with "real world" problems as well as students of yesteryears. In point of fact, the "real world" very seldom presents the pure computational tasks of the drill and practice type. The typical mathematical problem faced by the average adult involves first the translation of a practical situation into a number sentence. It is just this skill of "setting up" the appropriate algebraic relationships from a real life problem that is the focus of verbal problem solving.

This review of the literature on verbal problem solving in elementary mathematics includes: (1) The most frequently cited applied research studies; (2) Several methods for analyzing pupil mental processes; (3) A theoretical position seldom related to verbal problem solving; (4) Empirical attempts to apply this theory to verbal problem solving; and (5) A summary.

Applied Research

Suydam & Weaver (1977) have noted that most research on verbal problem solving has focused upon three major areas: (1) Characteristics of word problems; (2) Characteristics of successful and unsuccessful problem solvers; and (3) Teaching strategies that may help children become better problem solvers. Each of these research areas are briefly summarized below.

Characteristics of the Problem

Suydam & Weaver (1977) have summarized those verbal problem characteristics which appear to increase problem solving difficulty for most

pupils. Since these characteristics are cited in almost all such literature reviews, they are here listed as they appear in Suydam & Weaver review: (1) Data in a multi-step problem are not listed in the order needed for calculations; (2) There are irrelevant data included in the word problem; (3) There are no diagrams included in the problem presentation; (4) Essential data must be inferred from the problem; (5) Many steps are involved in the calculations; (6) Emphasis must be given to isolated word cues (such as "left" or "in all"); (7) Vocabulary is at or beyond pupil's level; (8) Content focus is unfamiliar to the students; (9) Computational solution is involved and tedious; and (10) The problem is stated without numerals (e.g. use of terms such as "half," "double," etc.).

The major problem characteristic not mentioned by Suydam & Weaver which has been found to pose a special difficulty for pupils is one identified by West (1977). West has discovered that word problems which do not allow the student to "count out" the answer are especially difficult for many individuals. Even in word problems where algebraic algorithms are the intended mode of solution, some students will engage in a lengthy counting process to obtain the correct answer. To assess the child's true mathematical skills, one must use numbers too large to allow for a counting solution.

Other than the vocabulary problem (number seven above) and the caution about counting solutions, problem characteristics which reliably cause student difficulties are characteristics which one may or may not include in a word problem, depending upon the skill that the teacher wishes to assess. It is important to know about these characteristics, but they should be eliminated from word problems only if the teacher is not interested in whether his/her pupils can ignore irrelevant data, deal

with unfamiliar situations, perform tedious calculations, etc.

Problem Solver Characteristics

As in other areas of student learning, there has been considerable research into what kinds of pupils perform best on word problems. "What are the characteristics of good problem solvers?"

Alexander (1960) and Hansen (1944) found that factors characteristic of high achievers in problem solving include: (1) the ability to recognize likenesses, differences and analogies; (2) an understanding of mathematical terms and concepts; (3) an ability to visualize and interpret quantitative facts and relationships; (4) good computational skills; (5) an ability to select procedures and data; and (6) comprehension of reading materials. Suydam & Weaver (1977) concluded that I.Q. is significantly related to good problem solver solving ability. They found that sex differences and socio-economic status alone were not relevant factors. Talton (1973) in a study of high and low achievers in verbal problem solving also concluded that I.Q. was the greatest contributor to success. This latter fact is especially discouraging to teachers of mathematics because raising student I.Q. is a difficult task. While there are studies which claim that I.Q. scores may be improved via various teaching programs, there is no indication that the native intelligence associated with successful problem solving in mathematics may be substantially modified by teachers in a typical classroom setting.

Jerman (1973) examined variables which contribute to problem solving ability. In grades four through six, the results suggested that certain computational variables played the dominant role in problem solving performance. At grade levels seven through nine, linguistic variables became more important and continued to be so into the college level. Smith (1971) made a comparison of the reading difficulty of sixth grade

problems on standard achievement tests. The conclusion was that a student's readability level may not be a principal reason for low scores on problem-solving achievement tests.

In a study with a group of high school and university students (Buswell & Kersh, 1956), tests and recordings were used to determine the thought processes as students worked on six sets of problems. Factors contributing to success in verbal problem solving were general reading ability, problem-solving reading skills, arithmetic factors, and spatial factors. Chase (1960) reports that the data from a study with sixth graders indicate that three primary factors are related to success in problem solving: computation, reading to note detail, and a knowledge of fundamental mathematical concepts.

One hundred fifteen sixth graders were involved in a study conducted by Robinson (1973). It was found that good problem solvers had significantly higher scores in the areas of I.Q., reading comprehension, arithmetic problem solving, and self-esteem. They had significantly lower scores on test anxiety. The more impulsive students were poor problem solvers while good problem solvers were the more reflective students. These traits were also found in a study by Walek (1973). Results of 63 high achievers and 63 low achievers from fourth grade showed that reflective students did significantly better than impulsive ones on a test designed to assess student ability in analyzing a problem and selecting the appropriate operation to solve it.

The essential conclusion to be drawn from the research into problem solver characteristics is that good problem solvers are generally those who possess the usual "good student" set of aptitudes, skills and personality traits. While this is hardly surprising, it is also not very enlightening. Perhaps this is one of the reasons the preponderance of

problem solving research has been in the area of teaching strategies.

Teaching Strategies

In 1961, Pace gave systematic instruction to a group of fourth graders to help them decide how to solve verbal problems and to help them understand why the process used was appropriate, e.g. analyze the problem. She found the experimental group made statistically significant gains over the control group which solved problems with no discussion. Pace's findings also indicate that gains in problem solving ability result if the children have an opportunity to solve many problems. Callahan & Glennon (1975) concluded that teachers of elementary school mathematics who give many verbal problems of appropriate difficulty will improve to some degree the ability to solve problems. In comparing these two studies, it appears that having students attend to the processes involved in verbal problem solving is more helpful than just having them solve a large number of problems.

VanderLinde (1964) experimented with the teaching of specific quantitative vocabulary and found it was effective in increasing problem solving scores. Studies by Hansen (1944) and Martin (1963) substantiate the fact that a study of mathematical vocabulary is pertinent to success in verbal problem solving.

Wilson (1964, 1967) conducted a study with fourth graders on one-step verbal problem solving using addition and subtraction problems. Two problem solving approaches were compared. One approach attempted to create a mental "set" focusing on the sequence of actions and events. The subjects were trained to recognize the action-sequence strategy of the problem, write an equation and then compute. The second approach attempted to create a mental "set" focusing on the "wanted-given" relationship.

This involved "seeing" the "wanted-given" relationship, expressing it in an equation, and then computing. The results of Wilson's study indicate that the "wanted-given" treatment group was superior in selecting the correct operation, getting the correct answer and in speed of solution. The same was true on a nine week retention test. Jerman (1973), however, did not find the "wanted-given" approach superior to other verbal problem solving approaches. It may be that particular strategies work best for particular pupils working on particular types of verbal problems. Certainly, no specific technique has been shown to be best for all students and all types of verbal problems.

A study by Zweng (1979) attempted to identify the things which help elementary school students solve verbal problems. The results of this study indicate that teaching children to make drawings holds promise. Drawing proved to be an effective "hint" and some children tended to use drawings more frequently if they had had one previous success in solving a problem with a drawing. Still an open question is: whether children can be taught to make drawings and then use this strategy on examinations without specific teacher direction and encouragement. Zweng also reviewed five popular sixth grade textbooks and reported that only two of the textbooks provided exercises for which children had to read drawings. None of the textbooks taught children to make their own drawings.

Sowder and Threadgill-Sowder (1980), in a study involving 262 fifth graders, presented problems with drawings and found this method more effective than a words-only presentation. Students interviewed about this approach to solving story problems indicated that the drawings were helpful.

Nelson (1974) completed a study with emphasis upon diagramming verbal problems. Three hundred sixty-two sixth graders were divided into

three instructional groups. One group solved problems presented in word form only. A second group practiced solving problems--some of which were presented in word form and some of which were presented in diagram form. This group received no instruction in diagramming word problems. A third group practiced the same categories of word problems as Group 2, however, Group 3 also received instruction in drawing diagrams to represent word problems and in translating diagram-posed problems to work form. (The instruction was based upon copying diagrams). The format of this study was the same for all three groups: pretest, eight instructional lessons, and posttest. Nelson's study indicated that the use of diagrams can be an effective tool in improving problem solving performance and the results suggest that students should be provided opportunities to solve problems presented in diagram form. Briefly, Nelson learned that diagram drawers (in sixth grade) are good problem solvers and that for some problems that lend themselves to diagramming, those instructed in diagramming/drawing do better than those not so instructed. Nelson also felt that more instructional sessions were needed to make a more noticeable difference. Teachers in his study drew diagrams to represent problem, and the students could copy them. The students were not actually instructed on how to draw their own.

Many authors express the opinion (some of which are data based) that children perform better in mathematics learning situations when diagrams/pictures are used. Lester (1980) refers to problem solving as "the process of coordinating previous experiences, knowledge and intuition in an effort to determine an outcome of a situation for which a procedure for determining the outcome is not known," i.e. the student does not have the understanding or the skills to proceed with solving such a problem as 2346 divided by 9. Lester states that students can be (and should be)

taught to be better problem solvers although the best method of teaching problem solving is not yet clear. In his list of 10 tips to teachers, Lester does suggest that pictures and diagrams are often helpful although sometimes confusing. He does not specify the procedure of teaching the student to draw his/her own diagrams in order to develop an image or feeling for the inherent structure of the problem.

At the first grade level Bolduc (1970) reports that students performed better on story problems if a picture or visual aid accompanied the reading of the story problem. Mark et al (1975) writes that drawing pictures or sketches for problems should start in the early grades because it helps the student to understand the situation. Neil (1969) reports that performance using diagrams was superior when he gave a group of third graders diagrams to be completed with half the problems. Sherrill (1973) and Webb & Sherrill (1974) found that accompanying a verbal problem with an accurate drawing generated better performance than the word statement alone.

As a strategy for problem solving, Polya (1957)suggests drawing a picture or diagram to represent the data of the problem. A visual approach is also emphasized by DeBono (1970). ". . . the advantage of drawing is that there is more commitment than with verbal explanation. Words can be general but a line has a definite place" (p. 282-283). Riedesel (1969) identified two strategies which increased problem solving ability--the use of mathematical sentences and the use of drawings and diagrams. In the latter case Riedesel is referring to drawings and diagrams provided by the teacher, not those generated by the pupils.

Collier and Lerch (1969) state that the use of objects, drawings or diagrams does make it easier for people of all ages and ability levels to solve many problems. They suggest that children be encouraged to use

these aids starting with the early grades and continuing throughout elementary school since these aids are effective in clarifying the problem, finding a solution and verifying the answer. Kramer (1970) suggests that the use of number lines and diagrams may be helpful in assisting the students to visualize the problem situation and thus improve problem solving ability. In a 1980 article, Driscoll recommends flexibility in representing problems to students since research has shown that children can better deal with problems which have visual aids or material available. He also recommends that children should be encouraged to manipulate the materials or make their own drawings or diagrams.

A brief review of the applied research literature on teaching strategies clearly shows that there are many specific techniques which have been shown to be helpful in assisting pupils to learn the complex skills of verbal problem solving. A possible approach for a teacher interested in this area is simply to consider the recommendations gleaned from literature by reputable authors such as Suydam & Weaver (1975, 1977), Suydam & Higgins (1977), Suydam & Dessart (1976) and Callahan & Glennon (1975).

Analyzing Mental Processes

There have been numerous attempts to measure what goes on inside Johnny's head while he is solving a verbal problem. One of the first to propose a comprehensive approach to this area of educational research was Brueckner (1935). He recommended: (a) observation of the pupil at work, (b) analysis of written work, (c) analysis of oral responses, and (d) interviews. Of these, having the student "think aloud" while attempting to solve a verbal problem appears to have become the method most frequently used by applied researchers. According to Callahan & Glennon (1975), however, classroom teachers are most likely to use day-by-day observations of

pupils, along with a subjective analysis of written work.

An article by Kilpatrick (1978) cited several methods which have been employed by applied researchers in assessing pupil mental processes during verbal problem solving. He notes that the early technique of asking the student to think aloud as he works has returned to favor as a result of the rise of information-processing psychologists and the use of this approach by contemporary Soviet researchers.

In reference to the technique of having students manipulate objects while solving a problem, Kilpatrick says, "But many interesting mathematical problems are not easily represented by means of an apparatus whose manipulation reveals the solver's mental processes" [p. 190]. Still, a number of researchers have employed manipulatives as teaching aids and as an inferential approach to the study of pupil mental processes. Many of these have been summarized by Suydam & Higgins (1977).

Grouws (1974) and others have used the interview in studying strategies used by children in solving verbal problems. This differs from the thinking aloud approach in that the pupil is asked to examine his thinking processes after attempting to solve a problem, rather than to provide a running commentary on what he is thinking while working. The interview method employs both specific questions by the researcher and general introspection by the student.

Introspection, on the part of researchers themselves, is likely the most ancient technique of all for gaining insight into the mental processes of verbal problem solving. While surely helpful, the major defect in this approach is that there are so many different mental strategies for arriving at both correct and incorrect solutions to most word problems. Wilson (1967) has suggested that even children with the same degree of general underachievement in mathematics present very different patterns of that

underachievement. It is also true that Piaget, Bruner and many others have cautioned against making the assumption that cognitive processes in children are the same as those in adults.

Despite a greatly increasing interest in the mental processes involved in verbal problem solving, there is as yet no clearly adequate technique for analyzing what is going on in Johnny's head as he works on such problems. The crucial need remains: an objective, reliable and valid diagnostic tool for determining wherein a particular pupil's attempt to solve a verbal problem has gone awry. More important still would be a technique for identifying general strategic deficiencies common among many students demonstrating difficulty with particular types of verbal problems in elementary mathematics.

Classroom Implications

While all of the approaches described above have made contributions to the understanding of pupil mental processes, none readily lends itself to use by a typical classroom teacher. Those which hold the most promise for research are precisely the ones least likely to be adopted by most elementary teachers. Few teachers have either the training or the time to employ these clinical diagnostic techniques with individual children.

A related challenge is that of the relationship between diagnosis and treatment. Even if a teacher were able to use one or more of the process analysis methods previously discussed, would his/her resulting data lead directly to a teaching technique? Too often, the answer is "No!" As noted in a prior section of this paper, most researchers have either recommended very specific teaching strategies (with little indication of how a teacher should appropriately select from among these innumerable strategies) or they have made such general suggestions that most

teachers respond, "I am already attempting to do that!" If even trained researchers are unable to translate their understandings of pupil mental processes into general teaching strategies not in common usage, it seems unlikely that classroom teachers would profit significantly from usage of the diagnostic tools so far developed.

One notable exception to the negative assessment presented above relates to the use of manipulatives. This approach to analyze pupil mental processes has led to a general recommendation for "activity-based" teaching/learning strategies. While most teachers are untrained in the use of manipulatives as diagnostic tools for individual pupils, they can (and some now do) implement activity-based learning. As noted by Suydam & Higgins (1977), the research in this area indicates that the approach is certainly promising.

Theoretical Considerations

Problem solving as a generic area of inquiry is at least as old as the writings of Plato. Philosophers and, more recently, psychologists have devoted a great deal of attention to this phenomenon. Surprisingly, very little of this work is clearly relevant to the process by which children attempt to solve verbal problems in elementary mathematics. There are, however, several themes in current psychological theory which may have implications for: (a) the diagnostic assessment of pupil mental processes; (b) verbal problem teaching techniques; and (c) heuristic strategies students may employ to improve their word problem achievement scores.

Intellectual Development

The seminal work of Jean Piaget (see, e.g., Ginsburg & Opper,

1969) has had a profound effect upon both educational theory and practice during recent decades. A closely related theoretical position has been promulgated by Jerome Bruner (see, e.g., Bruner, 1966). Both of these influential schools of thought hold that each child progresses through an invariant sequence of cognitive stages of intellectual development. The most general classification of these stages identifies three modes of learning potential: enactive, iconic and symbolic. The general notion here is that a child first develops ability to learn about his world via manipulation of physical objects (enactive stage). He then develops the capacity to manipulate mental images which represent the physical world (iconic stage). Finally the child acquires the ability to manipulate the abstract symbols of language, including mathematical symbols (symbolic stage).

While Piaget and Bruner have emphasized the importance of structuring each child's learning experiences within a mode appropriate to his intellectual stage of development, it may be important to follow the enactive-iconic-symbolic sequence <u>whenever</u> a new set of abstract concepts is to be learned--regardless of the child's developmental stage. It is, of course, possible to teach new concepts exclusively via the symbolic mode of instruction; but this does pose the risk that the pupil will simply learn to rotely manipulate the associated symbols without true understanding of the physical analogues being referenced. Indeed, this may be a major reason for the widespread student difficulty with verbal problems in mathematics.

The pupil who has rotely learned to perform algorithmic manipulations of algebraic symbols will be unable to correctly translate a word problem into the proper mathematical statement if he does not truly understand what is happening to the referenced physical analogues during

mathematical manipulations.

As previously noted in this literature review, there is a growing emphasis upon teaching elementary mathematics via the enactive mode (activity-based learning). Traditionally mathematics has been taught at the symbolic level. While some children likely use mental images to make the transition from enactive to symbolic learning, others may need more explicit help in this process than has heretofore been the norm.

Information Processing

Information processing theory has grown rapidly during the past two decades (see, e.g., Lindsay & Norman, 1972). This theoretical orientation has its roots in cognitive psychology and has been greatly influenced by computer science, especially recent attempts at computer simulation of human mental processes. Of particular relevance to the thesis of this paper are the attempts to simulate human problem solving via computer programs. The most referenced researchers in this area appear to be Herbert Simon and Allen Newell. In 1978, Simon won a Nobel Prize for his computer simulations.

One of the first books to discuss the impact of information processing upon the mainstream of problem solving research was published in 1966 (Kleinmuntz, 1966). One paper in this report of a 1965 symposium is entitled, "Cognitive Processes in Solving Algebra Word Problems" (Paige & Simon, 1966). A more recent publication (Chase, 1973) relates information processing theory to the specific focus of this paper: The role of mental imagery to solving mathematics word problems. Indeed, one paper in this text is entitled, "On the Function of Visual Imagery in Elementary Mathematics" (Hayes, 1973). While both of these books are highly technical, they do offer theoretical support for the notion

that mental imagery plays a crucial role in the mental processes involved in verbal problem solving.

Mental Images

In 1972, Yancey (1973) critically reviewed the existing theoretical and research literature relevant to the role of mental images in human thinking. This extensive literature review established the fact that almost all theories of human learning implicitly or explicitly point towards the importance of mental images in dealing with abstract concepts. Strangely, the Yancey dissertation was the first empirical investigation of the efficacy of graphical representations in the communication of abstract concepts. Even the technical term for such pictures of abstract concepts (ideographs) is still seldom used in the professional literature. Yancey found that while ideographs are quite common in science and mathematics texts, their function and value had not been studied prior to his research.

Summary

The importance of mental images in verbal problem solving may be directly or indirectly inferred from most contemporary theories of learning and instruction. The development of this specific application of theoretical considerations to elementary mathematics has been largely ignored. Text authors often use iconic representations of mathematical concepts, yet the efficacy of such usage has seldom been studied; and even then, the research is almost atheoretical.

Conceptual Imagery Research

In June, 1981, a computer search was conducted of the Psychological Abstracts data file using the official index terms "Conceptual Imagery"

and "Mathematics." There were 77 entries indexed by the first term and 3174 identified by the latter. The total number of documents indexed by both terms was zero! A second search was undertaken using the more generic term "Imagery," along with the quite general term "Mathematics." There were 2457 "Imagery" articles, yet only 21 which combined these terms.

Examination of the 21 documents listed in the Psychological Abstracts as "Imagery/Mathematics" yielded the information that only four had any relevancy to the issue of mental imagery in verbal problem solving. One of these was Hayes' chapter in Chase's book, cited in the preceeding section of this paper. The other three included two dissertations and one Romanian language journal article. Only the Romanian article focused upon problem solving in mathematics and an English translation was not available. The English abstract is discussed below. A search combining the terms "Iconic" (151 references) and "Mathematics" (3174 references) produced only three dissertations--none of which were relevant to verbal problem solving.

In June, 1981, a computer search was also conducted of the ERIC data file using the index terms "Imagery" and "Mathematics." There were 1083 documents identified under "Imagery" and 18,594 "Mathematics" papers. The total number of items coded under both terms was 16. Of these only six were even remotely related to mental imagery and verbal problem solving. Only one of the six papers was located in a journal generally available in most university libraries. This one document (Young, 1980) presents a brief literature based argument for the use of visual imagery in the conceptualization and solution of mathematical problems, but no data are offered.

The two computer searches cited above disclosed an extraordinary phenomenon. While there have been numerous papers written on mathematics

education and on mental imagery, only a very few have indicated the slightest connection between these two areas of theory and research. Almost all of these are doctoral dissertations or other unpublished papers; and none of these dealt with teaching students to generate their own diagrams as a mathematics problem solving strategy.

Despite considerable theoretical support for the notion that conceptual imagery plays a crucial role in verbal problem solving, there have been amazingly few attempts to research the phenomenon.

Empirical Studies

Of the four data-based studies identified by the two computer searches described above, three were doctoral dissertations. The following summaries of these studies are based solely upon their respective entries in Dissertation Abstracts.

Alexander (1978) tape-recorded individual interviews with 12 secondary school students about their conceptual difficulties with an initial algebra course. Analysis of the interviews led to an examination of the role of imagery in the conceptualization of algebra. Alexander concluded that the employment of mental imagery assisted students in developing symbolic means of manipulating the formulas correctly. Success in algebra, as measured by achievement tests in this study, was apparently more related to fluency in the use of imagery than to the employment of a purely symbolic approach.

Krakow (1979) studied 46 learning disabled boys of normal intelligence. One focus of the study was the relationship between figurative thought (perception, visual imagery, graphic imitation) and achievement in arithmetic. Figurative thought was measured via five published visual-spatial tests. Achievement was assessed with the Key Math

Diagnostic Arithmetic Test. The correlation between figurative composite scores and arithmetic achievement was $r = .66$.

Shoecraft (1972) employed three types of instruction of 12 classes of seventh graders ($n = 366$) and ten classes of ninth graders ($n = 336$). The control group was taught direct translation of algebra word problems into number sentences. One experimental group received visual representations (drawings) of the word problems being translated. The other experimental group was shown physical objects representing the word problems to be translated. On subsequent algebra word problem achievement tests (immediately after 12 days of instruction) the performances of the three groups were fairly comparable. The control group and the group receiving physical analogues of the instructional word problems did tend to perform better than the group receiving visual representations during the instructional phase. Shoecraft concluded that while the group receiving graphics did tend to pay more attention to the physical characteristics of each word problem than the other two groups, drawings did not appear to improve test performance. He stated that, "to expect such representation to facilitate mathematics learning is to assume that the mathematics implicit in the use of materials and drawings matches what is going on in the heads of students. The disparity between the two was evident in this study" (p. 3875-A). The subjects were not taught to generate their own diagrams.

The fourth empirical study located in the computer search was published in a Romanian language journal, and available only as an abstract in Psychological Abstracts. In that study, Bejat (1973) tested the role of imagery in verbal problem solving with 40 school children (13-14 years old) matched on intelligence and creativity test scores. There were ten children in each of four groups. The control group

studied verbal problem solving via a textbook. One experimental group received instruction in the use of graphics in verbal problem solving. A second experimental group received only graphic elaboration of word problems without explanation. In the other experimental group, the students were asked to generate their own graphic representations of word problems. On a subsequent verbal problem solving achievement test, two groups were superior to the others. One was the group requested to generate their own graphics and the other was the group which received graphic elaboration of word problems along with an explanation of the role of graphics. While this is the only previously reported assessment of the effects of teaching pupils to generate their own diagrams for solving word problems, the article is not available in the English language.

A manual search of the literature published prior to that included in the two data files searched via computer (1966 to mid-1981) yielded only one data based study relevant to this paper. In that study (Riedesel, 1964), five teaching strategies were combined and presented to 11 sixth-grade classes for ten weeks. Nine other classes were identified as the control group and they received their "normal" instruction. All experimental group students and all control group students were examined at the end of the ten-week period via a verbal problem solving test constructed for the experiment and by the problem-solving section of the Iowa Tests of Basic Skills. The mean experimental group score on each of these achievement tests was significantly higher than the respective control group means. One of the five instructional procedures used with the experimental group was that of presenting pupils with graphic representations of verbal problems. No attempt was made to separate the individual effects of the five "innovative" techniques employed in the study. It is also true that the research design and data analysis pro-

cedures used were quite primitive.

Suydam and Weaver (1975) do cite one other empirical study which may be relevant to the use of imagery/graphics in verbal problem solving. That is a dissertation by Swart (1970) in which fourth graders who were directed to draw a picture "telling the entire story" did as well on verbal problems as pupils who were taught to write equations to solve the problems. For area measurement problems, those who drew pictures scored significantly higher. Since the primary focus of the Swart study was a comparison of high and low structure in the development of problem solving ability, the use of student generated graphics--in lieu of traditional equations--has an uncertain relationship to the focus of this research.

Intuitive Recommendations

Despite the dearth of empirical data on the function and efficacy of imagery/graphics in verbal problem solving, it appears that intuitive support for the importance of graphics in mathematics education is quite strong. At least there is no paucity of papers which recommend the use of graphic representations in the teaching of verbal problem solving. The following is just a sampler of authors that have recently suggested this strategy: Suydam and Weaver (1975); Underhill (1977); Lovell (1971); Kennedy (1967); Kramer (1970); Marks, et al (1975); Jencks and Peck (1972); Riedesel (1967); West (1977); Suydam and Weaver (1977); and Greenes, et al (1977). One of the most explicit presentations of how to use graphics in building pupil mental imagery in mathematics is a book by Jencks and Peck (1968) entitled, Building Mental Imagery in Mathematics. This is a workbook containing many exercises intended to develop pupils' spatial-perceptual skills.

The intuitive but nonempirical support for the use of imagery/

graphics in mathematics problem solving still persists in the most recent publications by the National Council of Teachers of Mathematics. This is even the case for two books which certainly would be expected to contain data-based support for this teaching/learning strategy, if such research existed: Problem Solving in School Mathematics (Krulik & Reys, 1980); and Research in Mathematics Education (Shumway, 1980). One finds in each of these excellent books numerous, albeit implicit, words of encouragement for the thesis of the present study; but research support is nonexistent!

Summary

The literature on verbal problem solving in elementary mathematics is briefly summarized in the following listing of major findings.

1. While there have been thousands of articles on both conceptual imagery and verbal problem solving, none of these has clearly addressed both subjects in a data-based study.
2. Most learning theories suggest that conceptual imagery plays a crucial role in verbal problem solving, but empirical research is lacking.
3. Most lists of verbal problem solving teaching strategies recommend that teachers use diagrams to elicit appropriate mental images in their pupils, but they do not suggest that pupils be taught to generate their own diagrams.
4. No widely accepted empirical research has established the validity of the role of conceptual imagery/graphics in verbal problem solving, as suggested by both theory and mathematics education professors.
5. The intuitive support for the crucial role of conceptual

imagery/graphics in verbal problem solving appears to be so strong that empirical research has been ignored.

6. There is general agreement that verbal problem solving is both the most important and difficult to teach aspects of mathematics. There is also general agreement that a technique for the assessment of pupil mental processes is both the most crucial and most ignored aspect of mathematics education.
7. The only study in which the effects of student generated diagrams on an elementary mathematics word problem examination were assessed was reported in a Romanian language journal. The English abstract reports that the effect was significantly positive.

CHAPTER II

METHOD

This study was designed to assess the following general hypotheses.

1. Pupils who are taught to produce their own diagrams of the inherent structure of word problems in elementary mathematics will perform better on tests of word problem solving ability than pupils who are provided an equivalent amount of instruction in other problem solving strategies.
2. The diagrams will be exhibited by the subjects in the experimental groups (Method A) on both a posttest and a delayed retention test.
3. The pupils producing the most correct diagrams on a test will achieve a higher score on that test.
4. Pupils taught the diagramming strategy will exhibit a better attitude toward mathematics than pupils taught by traditional methods.
5. Less than 10 hours of instruction will be required to achieve the effects cited above.
6. Instructors unfamiliar with teaching pupils to generate their own diagrams will demonstrate an ability to successfully use this strategy.

Definition of Terms

Some of the terms employed in this paper are not widely used in the educational literature; others have multiple meanings. The following

definitions are used in this study.

1. Word problems: Quantitative situations presented to pupils in words and numerals so as to require the pupils to independently identify and perform appropriate arithmetical manipulations in order to arrive at the requested numerical answer. In this paper the term will always imply word problems in elementary mathematics.

2. Verbal problems: A term widely used in the literature to include word problems, but not used further in this paper because authors use the term to refer to non-mathematical test items.

3. Problem solving: A term that will not be used further in this paper without prior modification because it is frequently used in the psychological literature to denote a more generic mental process than intended here.

4. Word problem solving: Calculating the correct numerical answer for a word problem in elementary mathematics.

5. Correct diagram: The drawing of the inherent structure of a word problem which logically leads to a correct answer.

6. Inherent structure: A description (usually graphic) of the correct relationship among the quantitative components of a word problem.

7. Achievement test: In this paper the term applies only to tests containing problems and questions not previously presented to the pupils taking the tests.

8. Conceptual imagery: A mental picture of the inherent structure of a word problem.

Overall Design

Four intact fourth grade classes at Centerfield Elementary School, Oldham County School District, Kentucky, participated in this mathematics word problem solving study. The 92 subjects (23 per class) were given an attitude pretest, a pretest on solving mathematics word problems, six weeks of instruction on solving word problems (three 25 minute sessions per week), a posttest on solving word problems, an attitude posttest and a retention test on solving word problems (six weeks after the posttest).

Two of the fourth grade classes received Method A instruction while the other two groups received Method B instruction. Method A instruction was based upon a strategy of teaching the pupils to generate their own diagrams to represent the inherent structure of the word problems. Method B instruction was based upon an eclectic approach to solving word problems. The principal investigator (a first grade teacher at Centerfield School) taught two of the groups (one Method A and one Method B) while the Principal of Centerfield School taught the other two groups (one Method A and one Method B). The fundamental design was a 2 x 2 factorial, with instructional method as one factor and teacher as the second factor.

Subjects

Four of the five fourth grade classes attending Centerfield School were chosen for the study because of their heterogeneous groupings. A fifth fourth grade class was eliminated from the experimental portion of the study because one-third of that class constituted a homogeneous group of the highest achievers at this grade level. The students in this fifth class did take the three tests (pretest, posttest and retention test) since their scores provided a measure of the relative difficulty of each test. This fifth class also entered into some of the analyses and is referred to as the Reference Group.

Those students who were not present for numerous instructional sessions due to illness or involvement in special assigned classes, were not included in the study. They did participate in the instructional program on the days when they were in class. The number of regular participating students was 95. Three of these subjects were eliminated from the final analyses of data in order to equalize cell frequencies (see Data Analysis section below).

Testing Materials

Pretest, Posttest and Retention Test

All of the fourth and fifth grade students at the Crestwood Elementary School, Oldham County School District, Kentucky (five miles distant from the experimental school and having a similar student population) were administered a test consisting of 24 mathematics word problems. A total of 72 problems selected and modified from contemporary fourth and fifth grade mathematics textbooks had been randomly assigned to three separate tests (A, B and C). A list of textbooks consulted is listed in Appendix A. The District's adopted textbook for mathematics (Houghton Mifflin) was not used since the pupils would have access to these problems. These tests were distributed to the 231 students (one test per student) by their homeroom teachers. These tests were utilized to determine the psychometric characteristics of the individual problems, and to estimate the equivalence of forms and reliability of each form to be used in the study at Centerfield School. Twelve of the original 72 problems were eliminated because of poor psychometric characteristics. (See the Analysis section of Chapter II). The remaining 60 problems were randomly assigned (20 problems each) to the pretest, posttest and retention test to be used with the study subjects. Copies of these tests are included in Appendix B.

Attitude Test

Form B of the Suydam-Trueblood Attitude Toward Mathematics Scale (Suydam & Trueblood, 1974) was administered to the four participating classes during the week prior to commencement of the study and again after the posttest. This 26 item scale has a reported reliability (Cronbach's Coefficient Alpha) of .96 (Bechtel, 1969). A copy of the test is presented in Appendix C.

Standardized Achievement Tests

Two weeks after the commencement of the study, the annual standardized achievement tests were administered to all fourth graders by the Oldham County School Board (Comprehensive Tests of Basic Skills, Level 1, Form S, 1981). These standardized tests include subtests for mathematical and reading skills. Grade equivalence scores were recorded for each of the subjects for the following subtests: Mathematics Computation; Mathematics Application; and Total Reading.

Procedure

The study commenced at Centerfield School in the spring, 1981. The four participating fourth grade classes were administered the attitude pretest in their respective homerooms by the principal investigator. The following day all five fourth grade classes assembled in the School cafeteria for the word problem pretest. This test was administered by the principal investigator with explicit directions to the students, announced through an intercom system prior to the distribution of the test booklets. Each booklet consisted of five pages with four problems per page, allowing space for the students to show their work. Time allowed for the test was one hour.

The selection of Method A and Method B treatment groups was determined through the drawing of teachers' names. It was decided in advance of the drawing that the class of the first teacher drawn would receive Method A instruction. Since visual access from one room to another might be an important factor, the class diagonally across from the first class chosen for Method A treatment also became a Method A treatment group, leaving the remaining two classes for Method B instruction.

It was understood before the commencement of the study that during the 12 week period of the study the fourth grade teachers would follow the problem solving program presented in the District's adopted mathematics textbook (Houghton-Mifflin). These problems were not included in the study materials.

The same word problems and general procedures were used with both teaching Methods (A and B). All of the subjects in the program received a handout twice per week. The first two problems on the handouts were discussed orally at the beginning of each session and the remaining problems were worked individually and then solved at the chalkboard by volunteers. Handouts early in the program contained five problems each, but this number increased to eight as the subjects became more experienced at solving word problems. The third session each week was designated for small group involvement in writing problems, question and answer sessions, and dramatization of word problems. Part of the procedures for both Method A and Method B subjects included clarifying vocabulary and content, noting absences of essential information and/or presence of unnecessary information, and re-checking work assignments. The only difference in Method A and Method B instruction was one approach to solving word problems. Method A subjects were taught to draw a diagram in order to get a mental image of the inherent structure of each problem, while Method B subjects were taught by an eclectic approach as determined by strategies described in the literature on problem solving. These methods are described in detail later in this chapter.

Following the six week instructional period, all pupils in the four instructional groups, as well as those in the Reference Group, were administered the posttest. The same procedures used with the pretest were followed. The following day Groups A and B completed the attitude posttest.

This was the same form of the Suydam-Trueblood Attitude Toward Mathematics Scale employed as the attitude pretest. Six weeks following the posttest all study subjects (Groups A and B and the Reference Group) sat for the retention test. Administration procedures were identical to the pretest and posttest sessions.

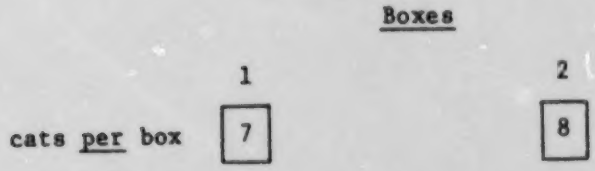
Instructional Materials

As previously noted, the same instructional problems and general procedures were utilized with both Group A and Group B. Each teacher followed the same lesson plan for his/her assigned classes. The only difference between the two instructional methods was the nature of the materials employed in teaching strategies for solving word problems.

Method A

Three different categories of diagrams were presented to Method A subjects during the six-week instructional period--tabular, geometric and contingency (see Appendix D). Lesson plans for the first week included very elementary problems, for example:

Marci had 2 boxes.
One box contained 7 cats.
The other box contained 8 cats.
How many cats did Marci have altogether?



7 + 8 = 15 cats altogether

Emphasis was upon diagramming the information within the problem in such a manner as to be able to visualize the inherent structure of the problem.

Early in the study addition was emphasized as the operation. The next step represented successive addition/multiplication. For example:

Mrs. Yancey has 2 gerbils.
Each gerbil has 7 baby gerbils.
How many baby gerbils in all?

Gerbils

	1	2
babies per gerbil	7	7

7 + 7 = 14 baby gerbils

or 2 x 7 = 14

This simple diagram technique was then expanded to three or more items.

There were 4 boxes.
7 fourth graders were hiding in each box.
How many fourth graders were hiding?

Boxes

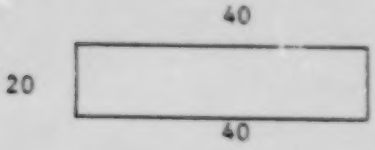
	1	2	3	4
students per box	7	7	7	7

7 + 7 + 7 + 7 = 28 students hiding

or 4 x 7 = 28

Problems requiring a geometric or contingency diagram technique were also taught throughout the instructional phase. An example of a geometric diagram follows.

A woman planned to put a wire fence around her garden. The garden is in the shape of a rectangle, 40 meters by 20 meters.
How much fencing will she need?



40
40
20
20
120 linear meters of fencing

A contingency diagram is shown below.

There are 45 houses on Madison Avenue. The houses are white, blue, yellow, green and gray. 18 of the houses are white. There are half as many blue houses as white houses. 15 of the houses are yellow. There is 1 gray house. How many houses are green?

Houses	Color
18	white
9	blue
15	yellow
1	gray
?	green
45 total	

45 - 18 - 9 - 15 - 1 = [2] green houses

Each week the problems became more difficult. The students in Method A were constantly encouraged to think through the problems as they diagrammed them in order to "get a picture in their heads." They were also encouraged to develop some of their own diagrams. Their efforts were accepted as long as the diagrams could be recognized as being descriptive of the inherent structure of the problem, i.e. an objective observer could identify the inherent structure by looking at the diagram. For example, the two diagrams following the next problem would be con-

sidered descriptive of the inherent structure of the problem being solved.

Mr. T has 2 barns.
There are 4 horses in each barn.
If each horse has 4 horseshoes, how many horseshoes are there altogether?

Barns

1 2

horses per barn

horseshoes per barn

16 + 16 = 32 horseshoes

OR

Barns

horses per barn

horseshoes per barn

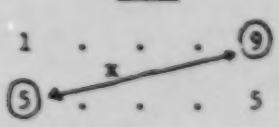
16 + 16 = 32 horseshoes

As the numerals increased in magnitude and frequency, the method of tabular diagramming became cumbersome. A shortcut system was introduced using suspension points (. . .).

Mary had 9 cats.
Each cat had 5 kittens.
How many kittens did Mary have altogether?

Cats

kittens per cat

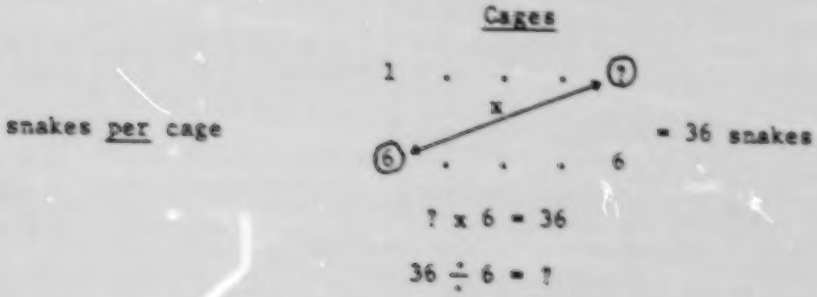


9 x 5 = 45 kittens altogether

The arrow indicates that the 5 occurs 9 times, allowing the students experiencing difficulty with multiplication to "see" the alternative method of successive addition to attain a correct solution.

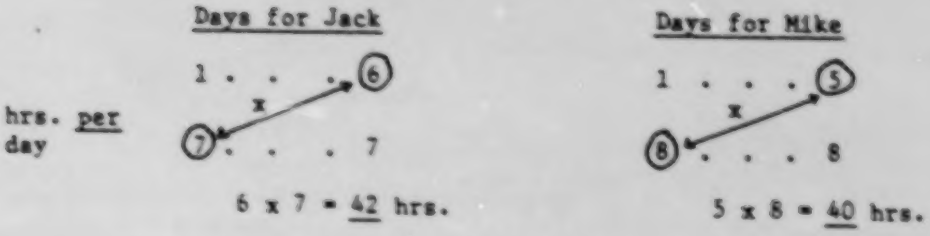
In order to show the relationship between multiplication and division, the following tabular type diagram was taught.

Bob had 36 snakes.
He put 6 snakes into each cage.
How many cages did he use?



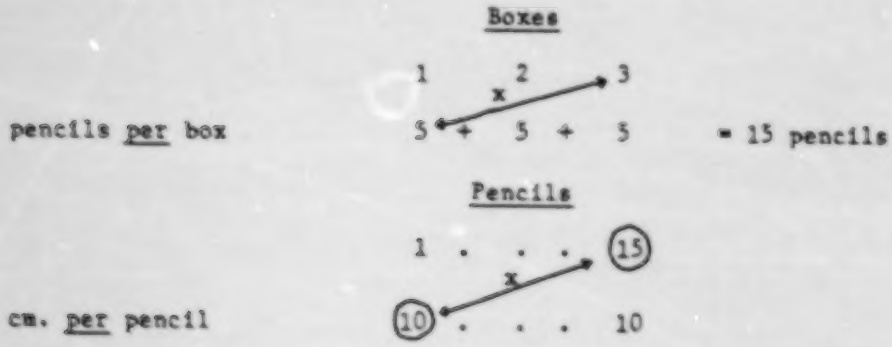
The last two weeks of the instructional phase concentrated on multi-step problems. Two examples of diagrams follow.

Jack worked 7 hours per day for 6 days.
Mike worked 8 hours per day for 5 days.
Which boy worked the most hours?



Solution: Jack worked 2 hours more than Mike.

Melissa had 3 boxes of pencils.
There were 5 pencils in each box.
Each pencil was 10 centimeters long.
How many centimeters altogether?



15 x 10 = 150 centimeters altogether?

Although the tabular type diagrams were stressed more than the geometric or contingency, examples of the other two diagrams were included on all the worksheets and discussed regularly. During the first three weeks of the instructional phase the handouts indicated to the students whether they should make a tabular diagram of the problem or simply "draw it" (referring to geometric/contingency diagrams). This "crutch" was eliminated during the last three weeks of the instructional phase (see Appendix E).

Alternative methods of drawing/tabling diagrams were discussed throughout the instructional phase of the study so that the students did not resort to memorization. The purpose of teaching diagramming as a problem solving strategy was to help the students explore the versatility of a tool/strategy which could be implemented if they were unsure of an algorithmic procedure to follow.

During the last two sessions of the instructional phase, the Method A subjects were given a variety of types of problems--a potpourri of the entire Method A instructional experiences. They were told during their

final instructional session that it was preferred that they use a diagramming technique on the posttest and retention test; however, if they felt completely confident of a correct solution when they read the problems, then they could move ahead and ration their time for problems which needed more thought--i.e. diagramming. Diagramming was described as a convenient tool/strategy for problems of a difficult nature. It was important that it not be thought of as an unnecessary chore.

Method B

For Method B, the traditional instructional methods of elementary mathematics were emphasized. These included computational practice, rule application and algorithm search. The method was expanded to include the procedures found in Appendix F, since it was thought to be the best teaching strategy not involving the drawing of diagrams. To the investigator, it was important that word problem solving be taught skillfully to all students participating in the study. In effect, Method B was an eclectic approach involving many different word problem solving strategies.

The format of lesson plans was identical to that used with Method A subjects. The only difference was the specific strategy emphasized. The use of drawing/diagramming problems was avoided in the instruction of Method B students. The instructors discussed each problem in the following terms: thinking through the problem to determine what was wanted and what was given, organizing the materials logically, viewing the problem on a smaller scale, thinking about the logical relationships among the quantitative components of the problem, asking if the answer seemed reasonable, etc.

Lesson plans for the first week included very elementary problems in order to establish a basis upon which to build. Each week the problems

increased in difficulty. As the numerals increased in magnitude and frequency, the students were taught to view the problem on a smaller scale--i.e. substitute smaller numbers for those in the problem. This tactic often elicited a quick recognition of the most appropriate computational procedure. In effect, this tactic seemed to elicit a clear mental image of the inherent structure of the problem (without diagrams). The students were also encouraged to complete a number sentence for each part of the problem and label their answers clearly. Some of these labeled number sentences were very similar to diagrams of the inherent structure.

Method B students were able to complete the handouts in less time than Method A pupils because they were not involved in diagrams/drawing, which was quite time consuming as a new learning experience. Because of this factor, the instructors altered the numerals and names in problems appearing on the handouts and gave more problems orally. This increased the number of problems Method B subjects received, but it did not change the lesson content because the skills of the extra problems were identical to those included in the lesson plans for any particular day.

It should be noted that since both instructors were teaching both Method A and Method B classes, and because they both knew that the study was focused upon diagramming, much of the Method B instruction was very similar to diagramming, without explicitly mentioning or using that strategy.

Test Scoring

The three tests (pretest, posttest and retention test) were scored by the researcher. The pupil received one point per problem if the precisely correct numerical answer was given in the space provided. The researcher also scored each test for the number of correct diagrams. A

correct diagram was defined as an accurate graphic representation of the inherent structure of a problem, with correct numerical entries and arithmetic operation symbols. See Appendix G for copies of pupil diagrams judged correct as well as examples of diagrams judged incorrect.

Data Analysis

Reliability Measures

Each of the three forms of the test (A, B and C) administered to the Achievement Test Development Group (see the Testing Materials section of Chapter II) was analyzed for the psychometric characteristics of individual problems and overall reliability. The analyses were performed separately for the fourth and fifth grade classes. The item analysis focused upon the difficulty index, point-biserial coefficient and *t* value for overall test means of successful and unsuccessful respondents (Guilford & Fruchter, 1978). The reliability indices calculated for each form of the test were the Kuder-Richardson formula 20 and the Average Item-Total Correlation (Guilford & Fruchter, 1978).

The twelve problems on the original tests (Forms A, B and C) demonstrating the weakest item analysis characteristics were discarded, and the remaining 60 problems were randomly assigned to the pretest, posttest and retention test. The same reliability indices calculated for the Achievement Test Development Group were computed for the study subjects on the pretest, posttest and retention test.

Elimination of Subjects

There were 23 pupils in each of the groups (one Method A and one Method B) taught by one of the instructors. The other instructor had 24 pupils in Method A Group and 25 in Method B Group.

When a factorial design has unequal cell frequencies, the analysis is greatly complicated. This is especially true if the subjects have not been randomly assigned to cells and a covariance correction is planned. Since both of these latter factors were true in this study, it was decided to create a fully orthogonal design by equalizing cell frequencies. Most statistics texts suggest a random elimination of subjects in this case (e.g., Snedecor & Cochran, 1967). In this study a much more conservative approach was employed. The three subjects chosen for deletion from data analysis (they had already participated in the study) were those with scores on the posttest closest to their respective cell means. This procedure left the cell means unaffected, but increased the cell variance. Of course, degrees of freedom and the statistical power associated with cell variance and degrees of freedom was reduced. This loss of data and statistical power was considered a reasonable price to pay for a straight forward orthogonal factorial analysis.

Induction Measures

To determine whether the Group A pupils had learned to independently generate correct diagrams for the inherent structure of mathematics word problems, each of the pretests, posttests and retention tests was scored for this variable. Total sums, means and standard deviations of correct diagrams were calculated. The group means (Method A vs. Method B/Reference Group) were so dramatically different that no further statistical analysis was needed to clearly establish the differential induction of this skill.

Pretreatment Group Differences

To assess the differences between the two instructional groups on the variables measured prior to commencement of the instruction, means

and standard deviations were calculated for the grade equivalence on the three standardized fourth grade achievement subtests used in the study: Mathematics Computation; Mathematics Applications; and Total Reading scores. Group means and standard deviations were also calculated for the pretest and attitude pretest. In all instances a simple t test was computed to determine the statistical significance of the mean contrasts. Because the cell means were equal and greater than 20, there was no need to assess the assumptions of normality and/or variance homogeneity in the respective populations (Games & Klare, 1967).

Posttreatment Group Differences

Cell and peripheral means and standard deviations were calculated for the 2 x 2 factorial design for the posttest, retention test and attitude posttest scores. For each of the three dependent measures a 2 x 2 factorial analysis of variance was computed.

Reference Group Performance

Means and standard deviations were calculated for the Reference Group on the pretest, posttest and retention test. To assess the improvement of the Reference Group from pretest to posttest and retention test, three dependent t tests were computed. Since two of these contrasts were nonorthogonal, the Tukey HSD procedure was used to maintain a single overall alpha for all three comparisons (Kirk, 1968). To secure a more precise statement of the obtained a posteriori significance level (to avoid multiple confidence limit estimates of population mean differences magnitude), the percentage points of the Studentized Range table were converted to regular t values via the $q/\sqrt{2}$ transformation (Kirk, 1968).

The same Tukey HSD procedure for dependent means was employed to

assess the improvement of Group A and Group B pupils from pretest to posttest and retention test. The improvement of all study groups was also expressed in terms of percentage improvement. The group differences on each test and their relative improvements were not statistically analyzed because most of these questions are addressed in subsequent analyses and because the unequal cell frequencies (Reference Group $n = 24$) would have necessitated a Behrens-Fisher t', with Welch df' correction (Kirk, 1968). This would have been an unnecessary exercise in light of the obvious mean and percentage differences and other analyses already planned.

Treatment Effects

The main treatment effects (instructional method, teacher and interaction) were assessed separately for each dependent variable (posttest, retention test and attitude posttest) via a standard 2 x 2 multivariable factorial analysis of covariance, with the five pretreatment measures as covariates (pretest, attitude pretest, mathematics computation test, mathematics application test and reading test).

The treatment effects were displayed more clearly in tables of adjusted means for each of the dependent measures. These means were adjusted via the deviation betas remaining after the effects of combined covariate regression had been introduced (Winer, 1971).

Characteristics of Diagrammers

Since the only pupils generating enough diagrams on the posttest and retention test to justify statistical analysis were from Group A, the following analyses were computed for Group A subjects only. Pearson product-moment correlation coefficients (Guilford & Fruchter, 1978) were calculated for the number of correct diagrams on the retention test. The same procedure was executed for the number of correct diagrams on the

retention test.

To assess the difference between posttest scores for those Group A pupils producing an above average number of correct diagrams on the post-test as contrasted with those generating an average or below average number of correct diagrams, a simple *t* test was employed. Although the number of subjects in each group was slightly different, the sample sizes and standard deviations were sufficiently similar to justify the regular *t*, as opposed to the Behrens-Fisher *t'* and Welch *df'* (Games & Howell, 1976). The same mean contrast was calculated via an analysis of covariance, using the five pretreatment measures as covariates. Analogous analyses were conducted for retention test correct diagram scores.

Sex Differences

The mean differences between males and females were assessed, via a standard *t* test, for all five pretreatment measures and three dependent measures. This procedure was executed within each of the four factorial cells, within each of the main effects and for overall differences. Again, the sample sizes and standard deviations were so similar that the regular *t* was justified (Games & Howell, 1976).

Tests of Statistical Models

As noted above, the experimental designs employed in this study eliminated the need to test the assumptions of the statistical models, except for the 2 x 2 multivariable factorial analysis of covariance (Nie, *et al*, 1975). Prior to each of the three analyses utilizing this model, the following four tests were calculated: the Cochrons C test for homogeneity of variance; the standard F test for combined covariate regression parallelism; and the standard F test for interaction of the main effects

(Snedecor & Cochron, 1967; Winer, 1971). These tests were computed separately for posttest scores, retention test scores and attitude post-test scores. A total of 42 statistical tests of the mathematical model were computed. Had any of these tests indicated serious violation of the model assumptions, it would have been necessary to modify accordingly the partition of sums of squares and degrees of freedom, as well as selection of error terms in the affected multivariable analysis of covariance (Hull and Nie, 1981). A significant interaction of main effects would have led to separate oneway analyses of covariance.

CHAPTER III

RESULTS

The data from this study have been organized into nine categories: (a) reliability measures; (b) induction measures; (c) pretreatment group differences; (d) posttreatment group differences; (e) reference group performance; (f) treatment effects; (g) characteristics of diagrammers; (h) sex differences; and (i) tests of statistical models. The findings related to each category are presented separately. The major findings are briefly summarized at the end of Chapter III.

Reliability Measures

The two principle dependent measures employed in this study (post-test and retention test) as well as the primary baseline measure (pre-test), developed specifically for this research, demonstrated the following psychometric characteristics.

Tables 1, 2 and 3 present the psychometric characteristics of the three forms of the test (Form A, Form B and Form C) administered to the Achievement Test Development Group. All forms of the test contained 24 items. The "K-R 20" refers to the Kuder-Richardson formula 20.

Table 1

Psychometric Characteristics of Form A in the Achievement Test Development Group

	Mean	Standard Deviation	n	K-R 20	Ave. Item-Total Correlation
Fourth Grade	11.03	6.52	39	.91	.72
Fifth Grade	12.58	6.17	38	.91	.73

Table 2

Psychometric Characteristics of Form B in the Achievement Test Development Group

	Mean	Standard Deviation	n	K-R 20	Ave. Item-Total Correlation
Fourth Grade	7.85	5.74	40	.91	.72
Fifth Grade	9.10	5.05	41	.87	.67

Table 3

Psychometric Characteristics of Form C in the Achievement Test Development Group

	Mean	Standard Deviation	n	K-R 20	Ave. Item-Total Correlation
Fourth Grade	9.74	5.86	38	.91	.74
Fifth Grade	9.37	5.27	35	.88	.70

The psychometric characteristics of the three tests (pretest, post-test and retention test) administered to the study subjects at Centerfield School are presented in Table 4. The study subjects include the 46 who received Type A instruction, the 46 receiving Type B instruction, the 24 pupils in the Reference Group and the three subjects dropped from further analysis. Analysis of individual groups disclosed no between group differences, except for test means. All tests contained 20 items. The 60 problems represented on these three tests were selected from the 72 problems used with the Achievement Test Development Group. All tests display sufficient reliability to detect effects large enough to be of

practical value.

Table 4

Psychometric Characteristics of the Pretest, Posttest and Retention Test

	Mean	Standard Deviation	n	K-R 20	Ave. Item-Total Correlation
Pretest	8.01	5.27	119	.91	.79
Posttest	12.14	4.02	119	.83	.66
Retention Test	12.01	4.66	119	.88	.75

Induction Measures

Since the only difference in the instruction provided to the two instructional groups (A and B) was the strategy for pupil generated diagrams of the inherent structure of word problems, Tables 5, 6 and 7 present the number of correct diagrams produced by each study group (A, B and Reference) on the pretest, posttest and retention test. There was clearly a differential inducement of correct pupil generated diagramming in Group A.

Table 5

Correct Diagrams Generated by Subjects on the Pretest

Instructional Group	total	n	Mean	Standard Deviation
A	0	46	0	0
B	0	46	0	0
Reference	0	24	0	0

Table 6

Correct Diagrams Generated by Subjects on the Posttest

Instructional Group	Total	n	Mean	Standard Deviation
A	280	46	6.09	4.02
B	0	46	0.00	0.00
Reference	0	24	0.00	0.00

Table 7

Correct Diagrams Generated by Subjects on the Retention Test

Instructional Group	Total	n	Mean	Standard Deviation
A	432	46	9.39	4.88
B	16	46	0.35	0.74
Reference	0	24	0.00	0.00

Pretreatment Group Differences

Since the instructional groups consisted of already existing (non-randomly assigned) fourth grade classes, Tables 8, 9, 10, 11 and 12 display five characteristics of the two groups of pupils prior to the commencement of instruction. Group A possessed lower scores than Group B on all of these measures. Three of these comparisons are statistically significant (Tables 8, 10 and 12).

Table 8

Grade Equivalence for Mathematics Computation Scores on the Standardized Fourth Grade Achievement Test

Instructional Group	n	Mean	Standard Deviation
A	46	5.25	0.81
B	46	5.87	0.80

$t = -3.69$, $df = 90$, $p < .001$

Table 9

Grade Equivalence for Mathematics Application Scores on the Standardized Fourth Grade Achievement Test

Instructional Group	n	Mean	Standard Deviation
A	46	6.04	1.55
B	46	6.64	1.88

$t = -1.67$, $df = 90$, $p < .05$

Table 10

Grade Equivalence for Total Reading Scores on the Standardized Fourth Grade Achievement Test

Instructional Group	n	Mean	Standard Deviation
A	46	6.86	2.02
B	46	7.70	2.01

$t = -2.00$, $df = 90$, $p < .05$

Table 11

Scores on the Attitude Pretest

Instructional Group	n	Mean	Standard Deviation
A	46	89.22	21.28
B	46	95.41	18.01

$t = -1.51$, $df = 90$, $p > .05$

Table 12

Scores on the Mathematics Word Problem Pretest

Instructional Group	n	Mean	Standard Deviation
A	46	6.52	3.90
B	46	8.61	5.11

$t = -2.21$, $df = 90$, $p < .05$

Posttreatment Group Differences

Tables 13, 14 and 15 present the performance of the two instructional groups on the three primary dependent measures. The tables are cross classified by the two teachers presenting each type of instruction. Each cell represents 23 pupils. None of the mean differences are statistically significant ($p > .05$). There are also no significant main effects or interactions ($p > .05$).

Table 13

Posttest Scores for Instructional Groups

Teacher	Instructional Group A	Instructional Group B	Total
C M	12.65	11.57	12.11
(SD)	(3.59)	(4.75)	(4.20)
D M	12.96	12.57	12.76
(SD)	(3.42)	(3.10)	(3.23)
Total M	12.80	12.07	12.43
(SD)	(3.47)	(4.00)	(3.74)

Table 14

Retention Test Scores for Instructional Groups

Teacher	Instructional Group A	Instructional Group B	Total
C M	12.52	11.17	11.85
(SD)	(3.34)	(5.07)	(4.30)
D M	12.30	13.39	12.85
(SD)	(3.08)	(4.48)	(3.84)
Total M	12.41	12.28	12.35
(SD)	(3.18)	(4.86)	(4.09)

Table 15

Attitude Posttest Scores for Instructional Groups

Teacher	Instructional Group A	Instructional Group B	Total
C M	99.04	97.78	98.41
(SD)	(24.07)	(18.47)	(21.22)
D M	97.35	93.65	95.50
(SD)	(20.41)	(19.53)	(19.84)
Total M	98.20	95.72	96.96
(SD)	(22.08)	(18.91)	(20.48)

Reference Group Performance

While not involved in either method of instruction, the Reference Group was administered the pretest, posttest and retention test to secure an indication of the relative difficulty of these measures. The performance of those pupils is displayed in Table 16.

Table 16

Reference Group Test Performance

	Pretest	Posttest	Retention Test
Mean	10.33	11.04	10.79
SD	5.64	4.30	5.84
n	24	24	24

In view of the differential improvement from pretest to posttest and retention test by each of the three study groups (Method A, Method B and Reference) a posteriori comparisons were calculated for each group. As noted in Chapter II, the Tukey HSD procedure for multiple dependent mean contrasts was employed. These data are displayed in Table 17.

Table 17

Relative Performance of Study Groups on the Pretest, Posttest and Retention Test

Study Group	Pretest	Posttest	Retention Test
A	6.52	12.80	12.41
B	8.61	12.07	12.28
Reference	10.33	11.04	10.79

Note: Row means which share underlines are not significantly different ($p > .50$). Row means which do not share underlines are significantly different ($p < .001$).

Beyond the factor of statistical significance is the simple magnitude of improvement in each of the study groups. These data are contained in Table 18.

Table 18

Percentage Improvement of Study Groups from Pretest to Posttest and Retention Test

Study Group	Posttest	Retention Test
A	96%	90%
B	40%	43%
Reference	07%	04%

Treatment Effects

As noted in Chapter II, the primary data analysis involved a 2 x 2 factorial analysis of covariance for each of the three dependent measures. The type of instruction (A and B) and the teacher (C and D) constituted the main effects. Because the four classes involved in the study were not equal in potentially relevant characteristics prior to treatment (see Tables 8, 9, 10, 11 and 12), those five variables were employed as covariates. The data from the three multivariable analysis of covariance procedures are presented in Tables 17, 18 and 19.

Table 19

Analysis of Covariance for Posttest Scores

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	Sig. of F
Within Cells	337.30	83	4.55		
Covariate Regression	870.18	5	174.04	38.29	.000
Constant	5.41	1	5.41	1.19	.278
Instruction	98.92	1	98.92	21.76	.000
Teacher	0.50	1	0.50	0.11	.741
Instruction x Teacher	0.19	1	0.19	0.04	.839

Table 20

Analysis of Covariance for Retention Test Scores

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	Sig. of F
Within Cells	606.93	83	7.31		
Covariate Regression	854.46	5	170.89	23.37	.000
Constant	4.70	1	4.70	0.64	.425
Instruction	50.72	1	50.72	6.94	.010
Teacher	2.07	1	2.07	0.28	.596
Instruction x Teacher	10.32	1	10.32	1.41	.238

Table 21

Analysis of Covariance for Attitude Posttest Scores

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	Sig. of F
Within Cells	18249.43	83	219.87		
Covariate Regression	19563.88	5	3912.78	17.80	.000
Constant	718.19	1	718.19	3.27	.074
Instruction	1140.37	1	1140.37	5.19	.024
Teacher	290.60	1	290.60	1.32	.254
Instruction x Teacher	70.58	1	70.58	0.32	.573

In the case of each dependent measure, the combined effect of the five covariates was the most powerful predictor of unexplained variance. In all analyses, however, there was a significant effect from the type of instruction. In no instance was there a significant teacher effect

or interaction between teacher and teaching method.

The type of instruction effect is most clearly seen when the dependent measures are adjusted for the influence of the five covariates. These data are presented in Tables 20, 21 and 22. (Tests of statistical significance are the same as those presented in Tables 19-21).

Table 22

Posttest Group Means Adjusted for Covariates

	Instructional Group		
Teacher	A	B	Total
C	13.59	11.43	12.51
D	13.53	11.19	12.36
Total	13.56	11.31	12.44

Table 23

Retention Test Group Means Adjusted for Covariates

	Instructional Group		
Teacher	A	B	Total
C	13.34	11.05	12.19
D	12.96	12.04	12.50
Total	13.15	11.54	12.35

Table 24

Attitude Posttest Group Means Adjusted for Covariates

Teacher	Instructional Group A	B	Total
C	101.70	95.86	98.78
D	99.85	90.42	95.13
Total	100.77	93.14	96.96

Characteristics of Diagrammers

As indicated by Tables 5, 6 and 7, only the pupils presented Type A instruction generated a significant number of correct diagrams for the inherent structure of the word problems on the posttest and retention test. Within the A Group, there was a significant correlation between the number of correct diagrams on the posttest and the total score on that test ($r = .36$, $df = 44$, $p < .02$). The correlation between correct diagrams on the retention test and number of correct answers on that test was even higher ($r = .66$, $df = 44$, $p < .001$). Both of these scatterplots showed excellent linearity and homoscedasticity.

There were no significant correlations between the number of correct diagrams on the posttest and the Standardized Fourth Grade Achievement Test scores: Grade equivalence for mathematics computation scores ($r = .10$, $df = 44$, $p > .51$); grade equivalence for mathematics applications scores ($r = .20$, $df = 44$, $p > .18$); grade equivalence for total reading scores ($r = .10$, $df = 44$, $p > .50$). There were significant correlations between the number of correct diagrams on the posttest and the other two pretreatment measures: Scores on attitude pretest

($r = .37$, $df = 44$, $p < .02$); scores on word problem pretest ($r = .35$, $df = 44$, $p < .02$). The analogous correlations between number of correct diagrams on the retention test and the pretreatment measures were all significant: Grade equivalence for mathematics computation scores on Standardized Fourth Grade Achievement Test ($r = .30$, $df = 44$, $p < .05$); grade equivalence for mathematics applications scores on Standardized Fourth Grade Achievement Test ($r = .32$, $df = 44$, $p < .03$); grade equivalence for total reading scores on Standardized Fourth Grade Achievement Test ($r = .41$, $df = 44$, $p < .004$); scores on attitude pretest ($r = .33$, $df = 44$, $p < .03$); scores on word problem pretest ($r = .48$, $df = 44$, $p < .001$). The correlation between the attitude posttest scores was significant in the case of correct diagrams on the posttest ($r = .33$, $df = 44$, $p < .03$), but not in the case of correct diagrams on the retention test ($r = .16$, $df = 44$, $p > .29$). Despite the differences noted above, the correlation between the number of correct diagrams on the posttest and number of correct diagrams on the retention test was clearly significant ($r = .65$, $df = 44$, $p < .001$).

The 22 pupils in the A Group who generated an above average number of correct diagrams on the posttest (more than six correct diagrams per pupil) attained a mean posttest score of 13.91. The 24 pupils who produced an average or below average number of correct diagrams (six or fewer correct diagrams per pupil) had an average posttest score of 11.79. This was a significant difference ($t = 2.15$, $df = 44$, $p < .04$). When the five pretreatment measures were used to adjust these two means via an analysis of covariance, the comparison was not significant ($F = 0.72$, $df = 1,39$, $p > .40$).

The 21 pupils in the A Group who generated an above average number

of correct diagrams on the retention test (more than nine correct diagrams per pupil) attained a mean retention test score of 14.52. The 25 pupils who produced an average or less than average number of correct diagrams (nine or fewer correct diagrams per pupil) had an average retention test score of 11.36. This was a significant difference ($t = 3.43$, $df = 44$, $p < .002$). When the five pretreatment measures were used to adjust these two means via an analysis of covariance, the comparison remained significant ($F = 9.14$, $df = 1,39$, $p < .004$).

Sex Differences

The only remaining categorical variable examined in this study was the sex of the pupils in each group. The number of females and males in each group is presented in Table 25.

Table 25

Number of Females and Males in Study Group

Sex	Group A	Group B	Reference Group	Total
Female	26	22	11	59
Male	20	24	13	57
Total	46	46	24	116

There were no significant sex differences ($p > .05$) on any of the five pretreatment measures or three posttreatment measures.

Tests of Statistical Models

As noted in Chapter II, the experimental designs employed in this study eliminated the need to test the assumptions of the statistical

models, except for the 2 x 2 factorial analysis of covariance. Prior to each of the three analyses utilizing this model, the following four tests were calculated: the Cochrans C test for homogeneity of variance; the Bartlett-Box F test for homogeneity of variance; the standard F test for combined covariate regression parallelism; and the standard F test for interaction of the main effects.

The homogeneity of variance tests required the calculation of 36 statistics, two tests for each of the five covariates and one dependent variable in each of the three utilizations of the 2 x 2 factorial analysis of covariance model. In 33 of these tests the null hypothesis of homogeneity of variance was retained at an alpha of $> .05$. The Bartlett-Box F statistic was significant ($.04 < p < .05$) for the attitude pretest scores in each of the three analyses. These data clearly support the legitimacy of the statistical model employed.

The test for combined covariate regression parallelism involved testing the significance of the interaction between the combined covariate effect and the main factorial effects, as well as the interaction of main effects. The results of these analyses are presented in Tables 26, 27 and 28.

Tables 26, 27 and 28 indicate that the null hypothesis of combined covariate regression parallelism for each cell of each analysis may be retained at an alpha of $> .05$. This allows the legitimate use of the pooled within-treatment combined covariate regression coefficient in the analyses presented in Tables 19-24.

The only other statistical model test involved the main effects interaction. As indicated in Tables 19-21 and 26-28, this effect was never significant.

Table 26

Analysis for Combined Covariate Regression Parallelism in Posttest Analysis of Covariance

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	Sig. of F
Within + Residual	298.97	68	4.40		
Constant	14225.39	1	14225.39	3235.54	.000
Covariates					
Math Comp. G.E.	157.30	1	157.30	35.78	.000
Math Appl. G.E.	405.65	1	405.65	92.26	.000
Reading G.E.	60.78	1	60.78	13.82	.000
Pretest	149.42	1	149.42	33.99	.000
Attitude Pretest	22.49	1	22.49	5.11	.027
Main Effects					
Instruction	98.96	1	98.76	22.51	.000
Teacher	0.52	1	0.52	0.21	.733
Interaction	0.19	1	0.19	0.04	.836
Combined Covariates by Main Effects & Interaction	78.33	15	5.22	1.19	.303

Table 27

Analysis for Combined Covariate Regression Parallelism in Retention Test Analysis of Covariance

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	Sig. of F
Within + Residual	466.14	68	6.90		
Constant	14027.13	1	14027.13	2046.27	.000
Covariates					
Math Comp. G.E.	212.77	1	212.77	31.04	.000
Math Appl. G.E.	469.72	1	469.72	68.52	.000
Reading G.E.	7.55	1	7.55	1.10	.298
Pretest	145.01	1	145.01	21.15	.000
Attitude Pretest	13.62	1	13.62	1.99	.163
Main Effects					
Instruction	50.63	1	50.63	7.39	.008
Teacher	2.32	1	2.32	.34	.562
Interaction	10.32	1	10.32	1.51	.224
Combined Covariates by Main Effects & Interaction	140.79	15	9.39	1.37	.188

Table 28

Analysis for Combined Covariate Regression Parallelism in Attitude Posttest Analysis of Covariance

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	Sig. of F
Within + Residual	14705.32	68	216.26		
Constant	864852.17	1	864852.17	3999.17	.000
Covariates					
Math Comp. G.E.	1943.69	1	1943.69	8.99	.044
Math Appl. G.E.	509.49	1	509.49	2.35	.129
Reading G.E.	163.66	1	163.66	.76	.387
Pretest	169.37	1	169.37	.78	.379
Attitude Pretest	15688.62	1	15688.62	72.55	.000
Main Effects					
Instruction	1090.68	1	1090.68	5.04	.028
Teacher	298.31	1	298.31	1.38	.244
Interaction	70.58	1	70.58	.33	.570
Combined Covariates by Main Effects & Interaction	3543.90	15	236.26	1.09	.380

All tests of the statistical assumptions of the mathematical models employed support the use of these models.

Summary of Results

The primary findings of this study are briefly summarized below. Statements 13, 14 and 15 are the major results of the study.

1. Both the posttest and the retention test were sufficiently reliable to detect effects large enough to be of interest.

2. There was a differential inducement of correct pupil generated diagramming as a mathematics word problem solving strategy in the study group.

3. Group A pupils possessed lower scores than Group B pupils on all five pretreatment measures; three of these mean comparisons were statistically significant (math computation test, reading test and pretest).

4. There were no statistically significant differences between Group A and Group B students on any of the dependent variable raw scores (posttest, retention test and attitude posttest).

5. The Reference Group demonstrated no significant improvement from the pretest to posttest or retention test.

6. Both instructional groups (A and B) showed improvements from pretest to posttest and retention test which were both statistically significant and substantial in terms of percentage improvement.

7. Analysis of covariance for each of the three dependent variables (posttest, retention test and attitude posttest) disclosed a significant instructional method effect, but no teacher effect or interaction.

8. For Group A pupils there was a significant correlation between the number of correct diagrams on the posttest and posttest performance. The same was true on the retention test.

9. Group A pupils producing an above average number of correct diagrams on the posttest scored significantly higher on the posttest than the less prolific diagrammers in Group A. The statistical significance of this difference was eliminated when the posttest scores were adjusted for the five pretreatment measures.

10. Group A pupils generating an above average number of diagrams on the retention test performed significantly better on that test than those producing fewer diagrams. This difference remained significant after covariance adjustment for the five pretreatment measures.

11. There were no sex differences.

12. Tests of all statistical models indicated no violations of assumptions serious enough to question the legitimacy of the models employed.

13. Pupils receiving instruction in diagramming as a mathematics word problem solving strategy (Group A) performed significantly better than those not receiving such instruction (Group B) on all dependent measures (posttest, retention test and attitude posttest).

14. Both Group A and Group B showed significant and substantial improvement in mathematics word problem solving ability during the period of the study; the Reference Group did not.

15. Within Group A, the pupils demonstrating the most proficiency in diagramming performed significantly better than those less competent in diagramming.

CHAPTER IV

DISCUSSION

All of the general hypotheses listed at the beginning of Chapter II were confirmed in specific detail by the results displayed in Chapter III. In light of the literature review presented in Chapter I, this study constitutes the first report of the effects of teaching pupils to generate their own diagrams of the inherent structure of mathematics word problems. The principle finding is that this skill can be learned, retained and used with considerable advantage on typical fourth grade elementary mathematics word problem tests.

Skill Acquisition

Since there have been no previously reported attempts to teach elementary school pupils to diagram the inherent structure of mathematics word problems, there is no basis of comparison for the success of the acquisition of such a skill in this study. The data in Tables 5-7 do clearly indicate that less than eight hours of instruction induces fourth grade students to produce a substantial number of correct diagrams on a posttest and on a delayed retention test. By the time of the retention test, the students taught to diagram problems were drawing as many correct diagrams on their tests as the more talented students in the Reference group were producing correct numerical answers.

Value of Skill Acquisition

Within the group of students taught to diagram word problems, those who best demonstrated that skill on the tests performed significant-

ly better on those tests than their Group A peers who produced fewer diagrams. While it may be true that many pupils could have produced correct diagrams but chose not to do so, it seems more likely that students who had acquired the skill tended to use that skill to correctly solve more problems. This conclusion is especially supported by the fact that adjusting retention scores for pretreatment measures (including pretest scores) still left a significantly positive effect for drawing an above average number of correct diagrams. The correlations between number of correct diagrams and number of correct answers on the posttest and retention test also suggest that those demonstrating better acquisition of the diagramming skill performed better, at least partly, because they had acquired the skill. It is also important to note that these above average diagrammers were being compared with others in the group that demonstrated the highest dependent measure scores and the most dramatic improvement in word problem solving achievement. It was a very difficult group within which to show a diagramming effect.

Value of Diagramming Instruction

The most important finding of this study was that teaching fourth graders to diagram mathematics word problems significantly improved their word problem solving skills. This is especially impressive in view of the performance of the group to which they were compared. Tables 17 and 18 indicate this phenomenon. When one recalls that it was the Reference Group which contained a group of the highest achievers at the fourth grade level, it is quite notable that Group A performed significantly better than Group B and Group B showed a great deal more improvement than the class containing a group of the highest achievers.

The fact that the experimental subjects demonstrated an average 67% improvement from pretest to retention test, while the Reference group showed only a 4% gain suggests that the instruction provided to the experimental subjects was clearly effective. One might suspect a simple Hawthorne motivational effect, but at least two factors argue against this as a complete explanation: (1) The Reference Group subjects were involved in the study to the extent of taking all three tests; and (2) There was no differential teacher effect in the analyses of variance. Pure motivational effects, especially of this magnitude, generally show one teacher to be more effective than another. If this was simply a student motivation phenomenon, doubling of test scores by exchanging classroom teachers within the same building for less than eight hours is truly noteworthy!

An alternative explanation for the substantial improvement of both instructional groups is that both groups learned a great deal about forming mental images of the inherent structure of mathematics word problems. As noted in Chapter II, the Method B instruction often came perilously close to teaching diagramming. With both instructors teaching both types of instructional groups, and both having just recently become involved in a new teaching/learning strategy (diagramming inherent structure) it is quite possible that even in the Group B classrooms the instructors were "reading" from their own mental images of the inherent structure and inadvertently (implicitly) communicating this to their pupils.

Although the dramatic improvement of both instructional groups (especially when compared to the Reference Group) greatly reduced the probability of an instructional effect, the Group A pupils did perform significantly better than Group B, when the clearly justified adjustment

for differential pretreatment scores was made. This was true for post-test performance, retention test performance and attitude posttest assessment.

The instructional method effect was not only statistically significant but also of substantial magnitude. The effect of instructional method accounted for 35% of the total variance in posttest scores and 90% of the variance left unexplained by the pretreatment measures (Table 19). The analogous percentages for the retention test were 21% and 68% (Table 20). Hence, whether the pupils received Method A or Method B instruction not only made a difference, it made a very large difference.

As indicated by Table 21, the instructional method accounted for 18% of the total variance in attitude posttest scores and 47% of the unexplained variability after the effects of the pretreatment measures were statistically removed. The method of instruction had both a significant and an important impact on pupil attitudes towards mathematics; a not unimportant consideration in selecting teaching techniques.

While it is quite clear that the diagramming method of instruction was superior to the Method B approach, it is also equally clear that this superiority would have been even more drastic had the diagramming group been compared with students of equal talent receiving truly traditional instruction. Even a direct comparison of the Group A pupils with the Reference Group would have produced much more impressive data than the presented contrasts with a group which received instruction quite similar to that of Group A.

Teacher Variable

The absence of a teacher effect and the absence of a teacher/method interaction not only simplified the data analyses and interpretations, it

also strongly suggests the transferability of both methods to other teachers and other classrooms. While the teacher effect was considered a fixed effect (vs. random effect) in the analyses of this study--i.e. the difference analyzed was between these two particular teachers, not among randomly selected teachers--the almost total absence of any difference between the two teachers is important. It is also noteworthy that both teachers were well known to the subjects (no excitement of "visitors from the University") and without prior experience in using either teaching method. Neither instructor had previously taught fourth grade pupils. One (the author) is a first grade teacher and the other (the Principal) has been an administrator for ten years. For both, the few hours of instruction for this study constituted a very small percentage of their respective responsibilities during the period of the study.

Pedagogical Implications

Of the data presented in Chapter III, the most instructive for teachers of elementary pupils may be those presented in Tables 17 and 18. In reviewing these tables, several points should be kept in mind: (1) As a group the study subjects' scores in mathematics computation, mathematics application and total reading (see Tables 8-10) averaged more than one grade above their actual grade level ; (2) The Reference Group contained a group of the highest achievers in the study school (see Subjects section of Chapter II), and they were receiving their regular instruction in solving mathematics word problems during the six weeks instructional phase of the study; (3) The instructional groups (A and B) received less than eight hours of instruction in solving mathematics word problems; (4) Method B pupils received instruction from teachers who

were teaching diagramming to Method A pupils; and (5) The teachers were well known to all study subjects, and all fourth grade pupils in the study school were participants in the study (less chance for a simple Hawthorne effect).

In view of the constraints noted above, it is quite surprising that the instructional groups so clearly surpassed the Reference Group in improvement from pretest to posttest and retention test (see Table 17). The fact that the Method A subjects also scored significantly greater than the Method B subjects (Tables 19-24) on all dependent measures should surely be of great interest to all elementary teachers. The doubling of pupil word problem solving skills (Method A) in less than eight hours of instruction is a notable achievement.

The pedagogical implications are clear: (1) Teach teachers to solve mathematics word problems by drawing diagrams and (2) Encourage teachers to communicate this innovative problem solving strategy to their pupils.

Further Research

While the data from this research are most encouraging for the use of student generated diagramming as a teaching technique in elementary mathematics, it is but the first of many inquiries into this innovative approach. In addition to simple replications, there is a need to answer numerous questions. Among these may be listed the following:

1. Would the method be of more or less benefit to pupils of more or less aptitude than those subjects in this study?

2. Would the effect be more or less pronounced if compared to truly traditional instruction rather than the Method A/Method B comparison used in this research?

3. Could the typical teacher adopt the techniques with such positive effects?

4. Would the technique be more or less effective at other grade levels?

5. Do the acquired pupil skills represent a permanent improvement in mathematics problem solving ability?

6. Would the technique be effective in such other areas of education as science, business mathematics, vocational education, etc.?

7. Could the technique be successfully taught to students by workbooks which show pupils how to diagram the inherent structure of word problems?

8. Which type of pupils would benefit most from which method of instruction?

9. Could student generated diagrams be used to diagnose individual pupil difficulties with mathematical concepts?

10. Does the inability to diagram a problem indicate that a student does not understand the concepts involved, even if he calculates a correct answer?

While the list of unanswered questions is essentially endless, it would appear that a very promising technique has been developed for the teaching of the most difficult area of elementary mathematics. Of potentially greater value is the use of this innovative technique as a valuable instrument of research.

CHAPTER V

SUMMARY

The professional literature contains numerous articles on conceptual imagery and numerous articles on mathematics education, but very few of these combine both topics in a data-based report. Most learning theories suggest that conceptual imagery plays a crucial role in solving mathematics word problems, but empirical evidence is lacking. Mathematics education articles that list strategies for teaching children to solve word problems recommend that teachers use diagrams to elicit appropriate conceptual images in their pupils, but they do not suggest that pupils should learn to generate their own diagrams.

In this study, four intact fourth grade classes (23 pupils each) were randomly assigned to one of four cells in a 2 x 2 factorial design. One factor was the type of instruction (Method A and Method B) and the other factor consisted of two teachers. The same mathematics word problems and general procedures were used with all groups for three 25 minute sessions per week over a period of six weeks. The difference in instruction was that Method A classes received instruction in the pupil generation of diagrams representing the inherent structure of mathematics word problems as a strategy for solving those problems. Method B pupils were taught an eclectic approach to solving word problems but no diagrams were used.

All pupils were administered three mathematics word problem tests: a pretest prior to commencement of instruction; a posttest at the end of the instructional period; and a retention text six weeks after the instructional period. A fifth fourth grade class at the same school was

also administered the same three mathematics word problem tests. This fifth class served as the Reference Group during the 12 week study period.

All pupils in the two instructional groups (A and B) also completed an attitude towards mathematics test prior to the instructional period and immediately following the period of differential instruction. In addition to the word problem pretest and the attitude pretest, three other pretreatment measures were recorded for the instructional groups: scores in reading, mathematics computation and mathematics application. These five pretreatment measures were collected as potential covariates in the analysis of posttreatment data.

The results showed that Method A pupils did learn to generate their own diagrams representing the inherent structure of mathematics word problems and they demonstrated this new skill on the posttest and retention test. Both instructional groups showed statistically significant improvement ($p < .001$) from the pretest to posttest and retention test. The fifth fourth grade class (the Reference Group) did not ($p > .50$).

Although both instructional groups (A and B) showed statistically significant improvement from pretest to posttest, a 2 x 2 factorial analysis of covariance, using the five pretreatment measures as covariates, indicated that the pupils taught to generate their own diagrams as an aid in solving word problems (Method A) performed significantly better than the Method B pupils on both the posttest and retention test ($p < .001$). They also expressed significantly better attitudes towards mathematics ($p < .02$). There was no teacher effect or interaction between the two factors.

Within the Method A group there was significant correlation between number of correct student generated diagrams on the posttest and

retention test, as contrasted with Method A pupils generating an average or below average number of correct diagrams.

This study has demonstrated the efficacy of teaching fourth grade pupils to generate their own diagrams representing the inherent structure of mathematics word problems. This was accomplished in less than eight hours of pupil instruction by teachers previously unfamiliar with the technique.

The implications of this study are: (1) Children can be taught to generate their own diagrams as a strategy in solving word problems; (2) The acquisition of this strategy improves their performance on word problem tests; and (3) Studying childrens' use of diagrams to solve word problems may be a useful instrument for diagnosing pupils mental processes as they develop skill in solving word problems.

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APENDIX A

LIST OF TEXTBOOKS

MATHEMATICS TEXTBOOKS

Holt School Mathematics
Holt, Rinehart & Winston, Publishers
New York
1978

Laidlaw Mathematics
Laidlaw Brothers Publishers
River Forest, Illinois
1978

MacMillan Mathematics
MacMillan Publishing Co., Inc.
New York
1976

Mathematics Around Us
Scott, Foresman & Co.
Glenview, Illinois
1978

Mathematics for Mastery
Silver Burdett Co.
Morristown, New Jersey
1978

Mathematics in Our World
Addison-Wesley Publishing Co.
Menlo Park, California
1978

APPENDIX B

PRETEST

POSTTEST

RETENTION TEST

PRETEST

NAME_____

(page 1)

Teacher_____

1. William saw 7 teams of dogs.
   There were 8 dogs on each team.
   How many dogs were there altogether?

Ans._____

2. If you have 8 dogs and 6 cats,
   how many legs do they have altogether?

Ans._____

3. Ralph collected 8 books of stamps.
   Each book had 6 pages.
   How many pages of stamps did Ralph have altogether?

Ans._____

4. Each of 5 children brought 3 hotdogs to a party.
   4 hotdogs were burned.
   How many were left?

Ans._____

Pre-T (page 2)

5. Mrs. Settle baked 28 cookies.
She gave 7 cookies to each child.
How many children got cookies?

Ans._____

6. A deck of cards contains 52 cards.
If the 4 Kings, 4 Queens and 4 Jacks are taken out of the deck,
how many cards will each player receive if the remaining cards
are dealt equally to 2 players?

Ans._____

7. Each seal gets 20 fish during each show.
If there are 2 seals,
how many fish will be needed for 3 shows?

Ans._____

8. If you need one (1) can of paint for 7 doors,
how many cans of paint will you need for 56 doors?

Ans._____

Pre-T (page 3)

9. A store put 80 hotdogs into 4 equal stacks.
How many hotdogs were placed in each stack?

Ans._____

10. A parking lot has room for 8 rows of cars with 9 cars parked in each row.
How many cars can be parked in the parking lot?

Ans._____

11. Jim has 3 records.
There are 5 songs on each side of a record.
How many songs does Jim have altogether?

Ans._____

12. There are 45 houses on Madison Avenue.
The houses are white, blue, yellow, green and gray.
18 of the houses are white.
There are half as many blue houses as white houses.
15 of the houses are yellow.
There is 1 gray house.
How many of the houses are green?

Ans._____

Pre-T (page 4)

13. Jenny needs to place 2 stamps on each envelop.
If she has a total of 58 stamps,
how many letters can she mail?

Ans._____

14. A ferryboat made 6 trips across the river.
Each trip carried 4 trucks and 7 cars.
How many motor vehicles did the ferryboat take across
the river altogether?

Ans._____

15. A rectangular floor is 30 feet long and 14 feet wide.
What is the perimeter?

Ans._____

16. Tom scored 2 points for each field goal and 1 point for each
free throw. He made 1 field goal and 3 free throws.
Ben scored 3 times as many points as Tom.
How many points did Ben score?

Ans._____

Pre-T (page 5)

17. A rectangular park measures 40 meters by 80 meters.
    If Sally walks around it twice,
    how many meters will she walk?

Ans._____

18. David lost some weight.
    Jean lost half as much weight as David did.
    His mother lost half as much weight as Jean lost.
    His mother lost 10 pounds.
    How much did David lose?

Ans._____

19. Joe worked in the garden 3 hours each day.
    He worked 5 days each week.
    How many hours did he work in 4 weeks?

Ans._____

20. There are 4 doghouses.
    3 dogs live in each doghouse.
    How many dog legs are there altogether?

Ans._____

POSTTEST

NAME_____

(page 1) Teacher_____

1. There are 18 gerbils in the pet store.
   The manager bought 2 more gerbils.
   He then sold 6 gerbils.
   How many gerbils were left?

Ans._____

2. Mr. Jones bought 8 cartons of soda.
   Each carton had 6 bottles.
   How many bottles of soda were there altogether?

Ans._____

3. Nancy put 4 apples in each bag.
   She had 28 apples altogether.
   How many bags did she use?

Ans._____

4. Susan has 6 fishbowls.
   She plans to put 7 fish in each bowl.
   How many fish does Susan plan to have?

Ans._____

Post T. (page 2)

5. Jim has 6 boxes of caterpillars.
There are 7 caterpillars in each of 3 boxes and 8 caterpillars in each of the other 3 boxes.
How many caterpillars does Jim have altogether?

Ans._____

6. A ferryboat only crosses the river when the boat is full.
The ferryboat is full when it holds 9 cars.
How many trips will the ferryboat have to make in order to transport 63 cars across the river?

Ans._____

7. Mr. Brown has a parking lot for trucks.
Each truck takes the space of three cars.
When there are 12 trucks in the lot, it is completely full.
If Mr. Brown used the lot for cars instead of trucks, how many cars would the lot hold?

Ans._____

8. Freda has 9 cats.
She has 3 times as many kittens as cats.
How many kittens does she have?

Ans._____

Post T (page 3)

9. John saw 9 cages of lions.
He saw 7 lions in each cage.
How many lions did he see altogether?

Ans._____

10. 4 buses are going to a baseball game.
48 students and 2 adults are going on each bus.
How many people will be going to the game on the 4 buses.

Ans._____

11. There are 48 lion paws.
They are equally divided into 3 caves.
How many lions are in each cave?

Ans._____

12. Tom has 3 shirts and 6 pairs of pants.
How many outfits does he have?

Ans._____

Post T. (page 4)

13. Candy comes in 2 pound bags.
One pound serves 20 people.
How many bags should be purchased in order to serve 200 people?

Ans._____

14. Relay teams were made from 2 classes.
Each class had 32 pupils.
If there are 8 pupils on each team,
how many teams were made?

Ans._____

15. There are 40 records.
Each set of 8 records costs $2.00.
How much will all 40 records cost?

Ans._____

16. The dining room at a hotel has 14 tables.
One-half of the tables have 6 chairs at each table.
The other half of the tables have 4 chairs at each table.
How many people can dine at the dining room at one time?

Ans._____

Post. T. (page 5)

17. The Pyramid of the Sun in Mexico is about 60 meters high.
Mica Dam in Canada is 4 times as high as the Pyramid of the Sun.
How high is the Mica Dam?

Ans._____

18. A ship has 70 passengers and some crewmen.
If all of the passengers and crewmen go to the lifeboats,
there will be 7 passengers and 2 crewmen in each lifeboat.
How many crewmen work on the ship?

Ans._____

19. 3 children shared 36¢ equally.
Then each child spent 5 ¢.
How much does each child have left?

Ans._____

20. Jack had 9 boxes of toys.
He had 4 toys in each box.
How many toys did he have altogether?

Ans._____

RETENTION TEST

NAME_____

(page 1)

Teacher_____

1. Beth has 11 pennies.
   She has 4 times as many nickels as pennies.
   How many nickels does she have?

Ans._____

2. There are 18 boys and 12 girls in Jim's class.
   If they form teams of 5 players each,
   how many teams can they have?

Ans._____

3. There are 42 steps altogether.
   There are 7 steps in each flight of stairs.
   How many flights of stairs are there?

Ans._____

4. There are 10 hotdog buns in each package.
   We need 70 buns for a picnic.
   How many packages of buns should we buy?

Ans._____

Retention (page 2)

5. There are 63 books.
They are packed equally into 9 boxes.
How many books are packed into each box?

Ans._____

6. Mrs. Yancey put 8 books on each desk.
She had 32 books altogether.
How many desks did she put books on?

Ans._____

7. 7 girls and 3 boys shared 50 marbles equally.
How many marbles did each child get?

Ans._____

8. 48 cub scouts were marching in a parade.
There were 4 columns of scouts.
How many scouts were in each column?

Ans._____

Retention (page 3)

9. One day the bike store sold 9 tricycles and 4 bicycles.
If each tricycle has 3 wheels and each bicycle has 2 wheels,
how many wheels left the store that day?

Ans._____

10. The fish in a large tank get 90 pounds of food each day.
How many pounds will they get in 7 days?

Ans._____

11. A building has 8 floors.
7 floors have 3 offices each.
The 8th floor has 4 offices.
How many offices are there altogether?

Ans._____

12. A woman is planning to put a wire fence around her garden.
The garden is in the shape of a rectangle, 40 meters
by 20 meters. How much fencing will she need?

Ans._____

Retention (page 4)

13. A cub scout troop hiked 7 miles in 2 hours.
If they continue at this rate, how many miles could the troop hike in 8 hours?

Ans._____

14. Jack took 4 rolls of film.
Each roll had 20 pictures on it.
8 pictures were of people.
How many pictures were not of people?

Ans._____

15. The Washington Monument is 9 meters taller than 3 times as high as the Leaning Tower of Pisa. The Leaning Tower of Pisa is 54 meters tall. How tall is the Washington Monument?

Ans._____

16. Mrs. Moore is buying Christmas presents for her 7 children to give to one another. If each child gives a present to each of the other children, how many presents must she buy?

Ans._____

Retention (page 5)

17. Jack has 6 comic books.
Each book has 4 pages.
Each page has 4 pictures on it.
How many pictures are there altogether?

Ans._____

18. Jack and Bill went fishing.
They each caught two kinds of fish--trout and pike.
Together they caught 18 trout and 18 pike. Bill caught 10 trout.
Jack caught 15 fish altogether.
How many pike did Bill catch?

Ans._____

19. A rectangular rug covers a floor space that is 12 feet by
21 feet. What is the distance around the rug?

Ans._____

20. As I was going to St. Ives, I met a man with 7 wives.
Every wife had 7 cats.
How many legs did I pass as I was going to St. Ives?
(animal and people)

Ans._____

APPENDIX C

ATTITUDE TEST

APPENDIX C

Attitude Toward Mathematics (Form B)

Marilyn N. Suydam & Cecil R. Trueblood
Pennsylvania State University

This is to find out how you feel about mathematics. You are to read each statement carefully and decide how you feel about it. Then indicate your feeling on the answer sheet by marking:

SA - if you strongly agree
A - if you agree
N - if you feel neutral
D - if you disagree
SD - if you strongly disagree

1.	Mathematics often makes me feel angry.	SA A N D SD
2.	I usually feel happy when doing math problems.	SA A N D SD
3.	I think my mind words well when doing math problems.	SA A N D SD
4.	When I can't figure out a problem, I feel as though I am lost in a mass of words and numbers and can't find my way out.	SA A N D SD
5.	I avoid mathematics because I am not very good with numbers.	SA A N D SD
6.	Mathematics is an interesting subject.	SA A N D SD
7.	My mind goes blank and I am unable to think clearly when working math problems.	SA A N D SD
8.	I feel sure of myself when doing mathematics.	SA A N D SD
9.	I sometimes feel like running away from my math problems.	SA A N D SD
10.	When I hear the word math, I have a feeling of dislike.	SA A N D SD
11.	I am afraid of mathematics.	SA A N D SD
12.	Mathematics is fun.	SA A N D SD
13.	I like anything with numbers in it.	SA A N D SD
14.	Mathematic problems often scare me.	SA A N D SD
15.	I usually feel calm when doing math problems.	SA A N D SD
16.	I feel good toward mathematics.	SA A N D SD
17.	Mathematics tests always seem difficult.	SA A N D SD
18.	I think about mathematics problems outside of class and I like to work them out.	SA A N D SD
19.	Trying to work math problems makes me nervous.	SA A N D SD
20.	I have always liked mathematics.	SA A N D SD
21.	I would rather do anything else than do math.	SA A N D SD
22.	Math is easy for me.	SA A N D SD
23.	I dread math.	SA A N D SD
24.	I feel especially capable when doing math problems.	SA A N D SD
25.	Mathematics class makes me look for ways of using mathematics to solve problems.	SA A N D SD
26.	Time drags in a mathematics lesson.	SA A N D SD

APPENDIX D

EXAMPLES OF DIAGRAMS

DIAGRAMS

Simple tabular diagram:

Mr. Thompson likes to smoke a pipe.
He has 2 piperacks.
There are 4 pipes in each rack.
How many pipes does Mr. Thompson have altogether?

	Piperacks	
	1	2
pipes per rack	4	4

4 + 4 = 8 pipes altogether

"Short cut" tabular diagram:

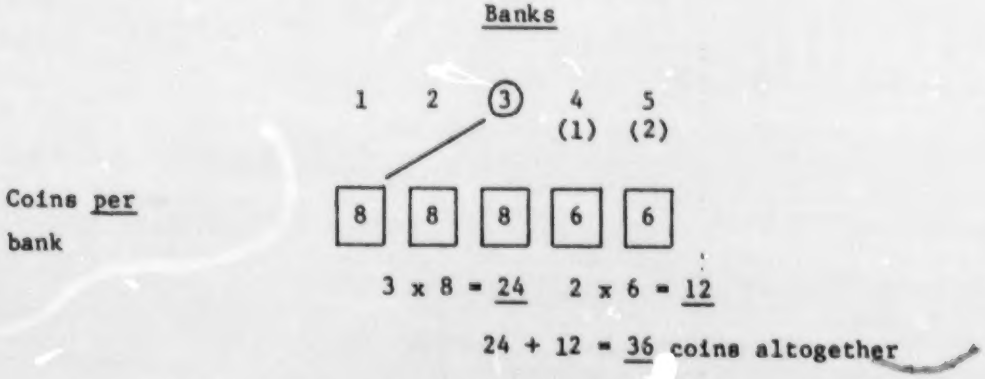
Mrs. Yancey has 9 trees in her yard.
There are 3 birdnests in each tree.
How many nests altogether?

	Trees					
	1	2	.	.	.	9
			x			
nests per tree	3		.	.	.	3

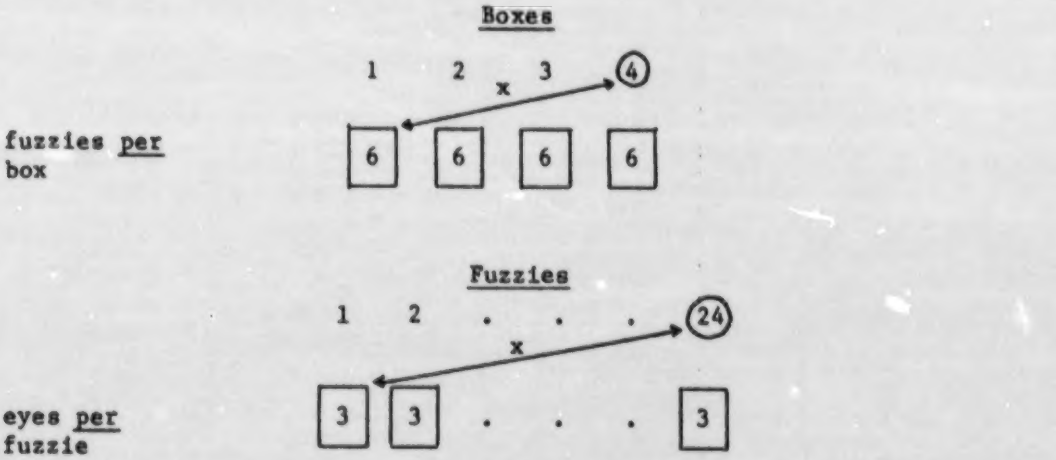
9 x 3 = 27 nests altogether

Complex tabular diagrams:

(1) Don collects coins.
He keeps them in 5 banks.
There are 8 coins in each of 3 banks.
There are 6 coins in each of 2 banks.
How many coins does Dan have altogether?

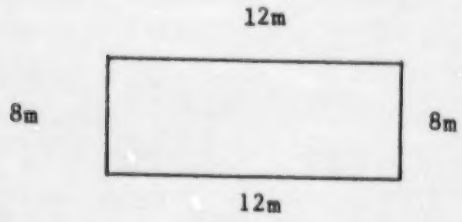


(2) Mrs. Ammons loves warm fuzzies.
She has 4 boxes of them.
She keeps 6 warm fuzzies in each box.
They each have 3 eyes.
How many eyes altogether?



Example of geometric diagram:

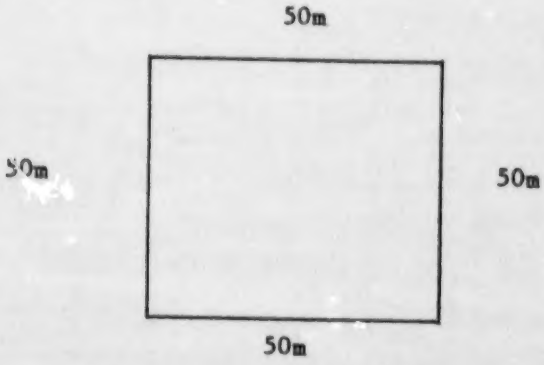
Jane has a rectangular swimming pool.
The pool is 8 meters wide and 12 meters long.
How many meters will Jane walk if she goes
around the edge of the pool once?



8 x 2 = 16m
12 x 2 = 24m
40meters

More complex geometric diagram:

There is a square park near the school.
One side is 50 meters long.
Mrs. Settle jogs around the park twice each day.
How many meters does she jog per day?



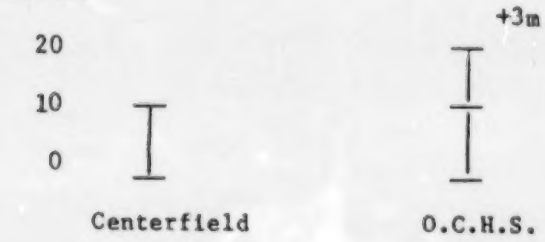
50 x 4 = 200 meters
x 2
400 meters

or

Successive addition

Contingency diagrams:

The Oldham County High School is 3 meters taller than 2 times the height of Centerfield School. If Centerfield School is 10 meters high, how high is Oldham County High School?



10 + 10 + 3 = 23 meters tall

Anna and Gayle went rock hunting.
They each found 2 kinds of rocks--
big and small.
Together they found 15 big rocks
and 15 small rocks.
Gayle found 8 big rocks.
Anna found 12 rocks altogether.
How many small rocks did Gayle find?

	Big	Small	Total
Anna	(7) ?	(5) ?	12
Gayle	8	(10) ??	(18) ?
	15	15	30

Solution: Gayle found 10 small rocks.

APPENDIX E

LABELED HANDOUT

SAMPLE OF LABELED HANDOUT FOR GROUP A

1. John work 5 days each week in the grocery store.
   He worked 8 hours each day.
   How many hours did he work altogether?

   Label_____

   Label_____

   Answer_____

2. Jack is 6 feet tall.
   Mark is half as tall as Jack.
   Mary is 2 inches taller than Mark.
   How tall is Mary?

   DRAW IT!

   Answer_____

3. Tommy Tucker sang for his supper.
   He sang 4 songs each day, 5 days each week.
   He sang for 10 weeks.
   How many songs did he sing altogether.

   Label_____

   Label_____

   Label_____

   Label_____

   Answer_____

APPENDIX F

INSTRUCTIONAL PROCEDURES

FOR METHOD B

INSTRUCTIONAL PROCEDURES METHOD B

Purpose: To provide an eclectic program with mathematical word problems at appropriate levels of difficulty.

1. Analyze the problem (think through).
2. Determine the wanted-given structure.
3. Note absence of essential information and/or presence of unnecessary data.
4. Translate problem into a number sentence.
5. Present problem orally and in written form.
6. Clarify any content focus which might be unfamiliar.
7. Define unfamiliar vocabulary if necessary.
8. Stress logical order for setting up multi-step problems.
9. Work numerous problems.
10. Emphasize the importance of re-reading and re-checking the problem and computation.
11. Look at the problem and view it on a smaller scale.
12. Dramatize the problem situation.

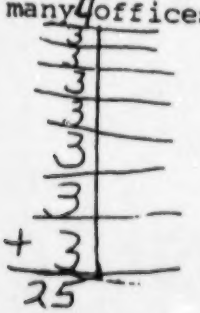
APPENDIX G

EXAMPLES OF PUPIL GENERATED DIAGRAMS

SCORED AS CORRECT/INCORRECT

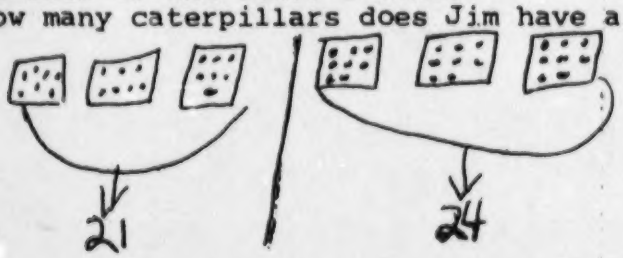
PUPIL GENERATED DIAGRAMS SCORED AS CORRECT

11. A building has 8 floors.
7 floors have 3 offices each.
The 8th floor has 4 offices.
How many offices are there altogether?



Ans. 25 offices

5. Jim has 6 boxes of caterpillars.
There are 7 caterpillars in each of 3 boxes and 8 caterpillars in each of the other 3 boxes.
How many caterpillars does Jim have altogether?



21
+24
45

Ans. 45

5. Jim has 6 boxes of caterpillars.
There are 7 caterpillars in each of 3 boxes and 8 caterpillars in each of the other 3 boxes.
How many caterpillars does Jim have altogether?

21
24
45

Ans. 45 cats.

10. The fish in a large tank get 90 pounds of food each day.
How many pounds will they get in 7 days?

90 + 90 + 90 + 90 + 90 + 90 + 90 = 630

Ans. 630 pounds

7. Mr. Brown has a parking lot for trucks.
Each truck takes the space of three cars.
When there are 12 trucks in the lot, it is completely full.
If Mr. Brown used the lot for cars instead of trucks,
how many cars would the lot hold?

1 2 3 4 5 6 7 8 9 10 11 12

cars per truck 3 3 3 3 3 3 3 3 3 3 3 3

12
× 3
36

Ans. 36 cars

9. One day the bike store sold 9 tricycles and 4 bicycles.
If each tricycle has 3 wheels and each bicycle has 2 wheels,
how many wheels left the store that day?

tricycles		bicycles	
1	9	1	4
Wheels 3	× 3	2	× 2
	27		8

27 + 8 = 35

Ans. 35 wheels

PUPIL GENERATED DIAGRAMS SCORED AS INCORRECT

7. 7 girls and 3 boys shared 50 marbles equally.
How many marbles did each child get?

Boys
1 2 3
? ? ? = 50

Girls
1 . . . ?
? . . . ? = 50

+? +3 10 5 10 50 Ans 5 marbles

16. Mrs. Moore is buying Christmas presents for her 7 children to give to one another. If each child gives a present to each of the other children, how many presents must she buy?

+ + + + + + +

Ans 20

17. Jack has 6 comic books.
Each book has 4 pages.
Each page has 4 pictures on it.
How many pictures are there altogether?

Books
1 . . . 6 + 6
1 . . . 4 + 4
1 . . . 4 14

Ans 14

3. Nancy put 4 apples in each bag.
She had 28 apples altogether.
How many bags did she use?

apples in each bag

1 2 3 4

28 28 28 28

$4\overline{)28}$ = 7

Ans. 7

17. The Pyramid of the Sun in Mexico is about 60 meters high.
Mica Dam in Canada is 4 times as high as the Pyramid of the Sun.
How high is the Mica Dam?

60 × 4 = 240

Ans. 240

9. One day the bike store sold 9 tricycles and 4 bicycles.
If each tricycle has 3 wheels and each bicycle has 2 wheels,
how many wheels left the store that day?

wheels

1	2	1	2	3
9	9	9	9	9
	18			27

27 + 18 = 45

Ans. 45 wheels

VITA

Anna Vance Yancey was born in Harrisonburg, Virginia on May 16, 1933. She was graduated from Harrisonburg High School in 1951. She received the degree B. S. cum laude in elementary education from Elizabethtown College (Pa.) in 1969, and a M.Ed. degree in elementary education from The Pennsylvania State University in 1972. In 1975 she entered the University of Louisville School of Education pursuing an Educational Specialist degree in elementary education.

Mrs. Yancey worked as a research assistant on numerous research grants established by the Office of Education, National Science Foundation and Office of Economic Opportunity from 1963 to 1969. Since 1970, she has been employed as a public school teacher in Pennsylvania and Kentucky. For the past six years she has served as a first grade teacher and team leader with the Oldham County School Board in Oldham County, Kentucky. She holds a Kentucky Rank I teaching certificate in elementary education and has published several research articles in national educational journals.